SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT203 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 1 - DISCRETE FOURIER TRANSFORM

TOPIC – LINEAR CONVOLUTION

LINEAR CONVOLUTION

- The convolution sum relates the input, output and unit sample response of the discrete time systems
- Linear convolution is a very powerful technique used for the analysis of Linear Time Invariant systems
- x(n) can be expressed as sum of weighted impulses

$$\mathbf{y(n) = x(n) * h(n)}$$

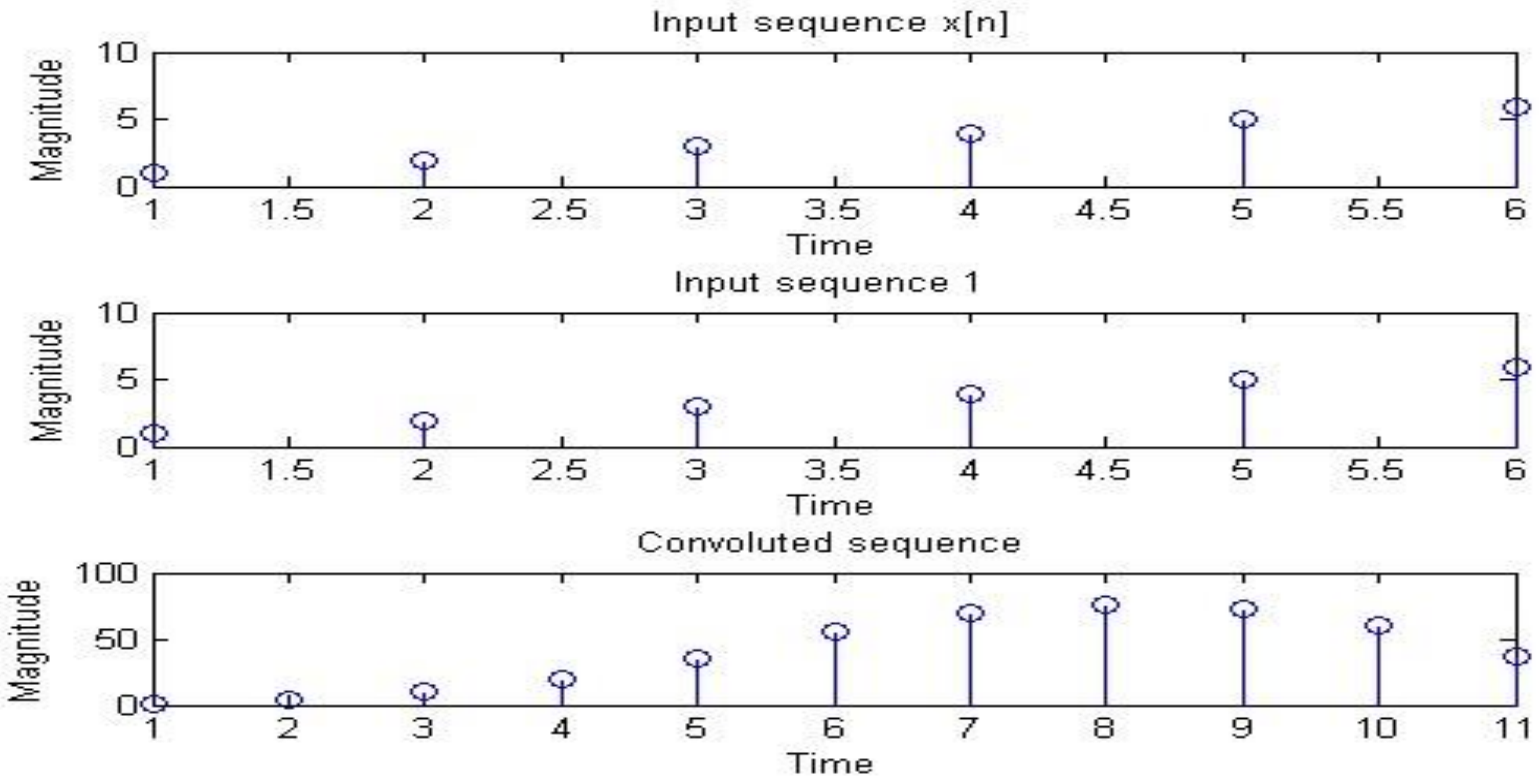
LINEAR CONVOLUTION

- The behavior of the LTI system is completely characterized by the unit sample response h(n)

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

- It is the linear convolution of x(n) and h(n) gives y(n) Inverse Z Transform:

REPRESENTATION OF CONVOLUTION



REPRESENTATION OF CONVOLUTION

$x(n)$ \ $x_1(n)$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$$y(n) = \{1, 4, 10, 20, 35, 56, 70, 76, 73, 60, 36\}$$

Four methods available to compute convolution sum:

1. Definition Method
2. Graphical Method
3. Tabulation Method
4. Multiplication Method

CONVOLUTION SUM

Four steps involved in computing convolution sum:

1. Folding

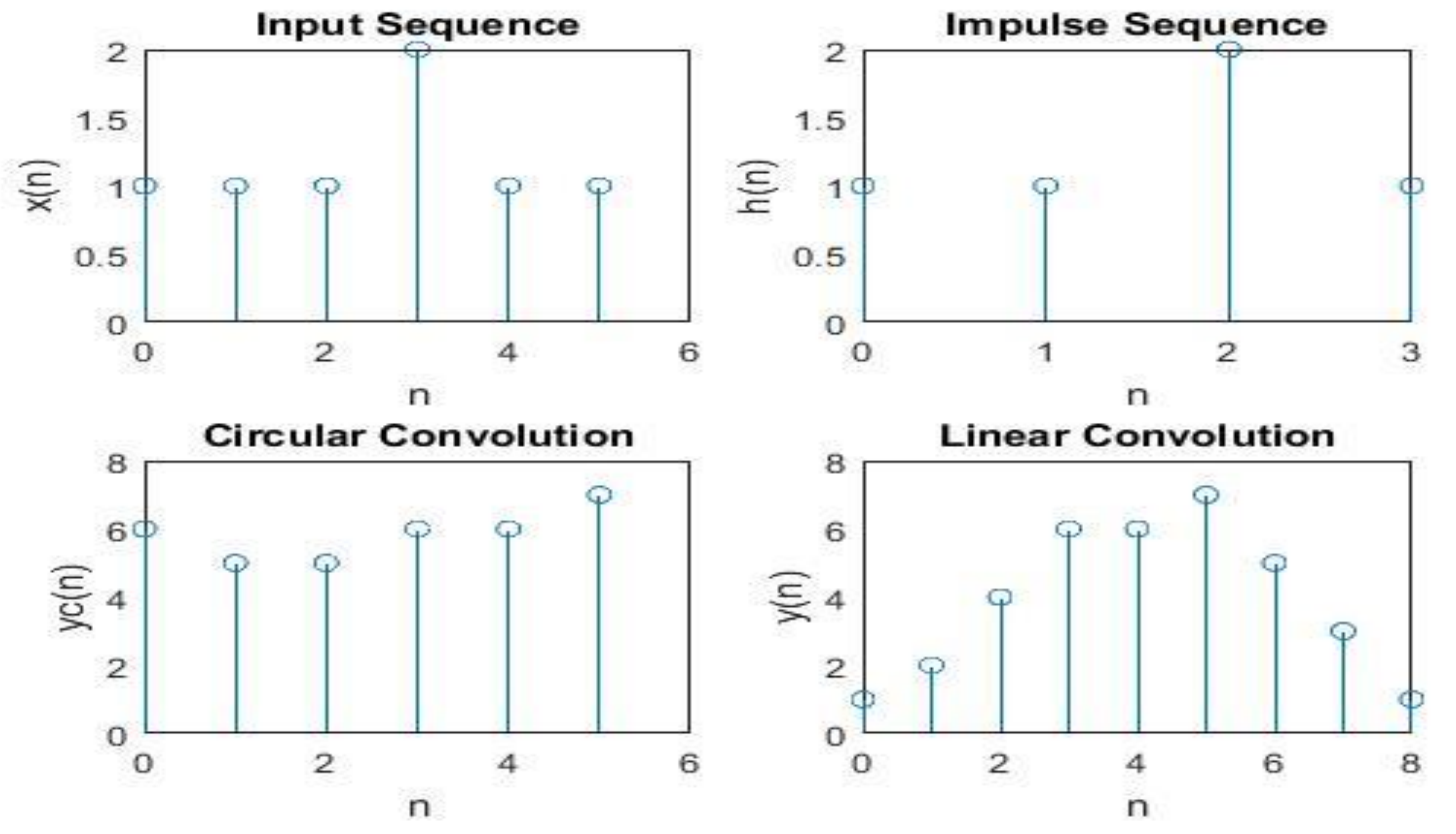
2. Shifting

3. Multiplication

4. Summation

- Let M be the total no. of samples of x(n) and N be the total no. of samples of h(n) then the total no. of samples in y(n) be **M+N-1**

CONVOLUTION SUM



CONVOLUTION SUM

$x(n) = \{1, 1, 1, 2, 1, 1\}$ $h(n) = \{1, 1, 2, 1\}$

			1	1	1	2	1	1
					1	1	2	1
			1	1	1	2	1	1
		2	2	2	4	2	2	
	1	1	1	2	1	1		
1	1	1	2	1	1			
1	2	4	6	6	7	5	3	1

$y(n) = \{1, 2, 4, 6, 6, 7, 5, 3, 1\}$

PROPERTIES OF CONVOLUTION SUM

- It can be classified into

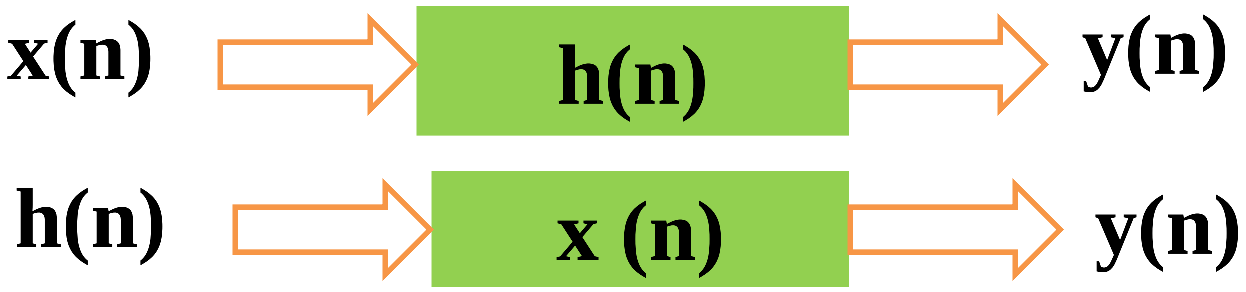
1. Commutative Property

2. Associative Property

3. Distributive Property

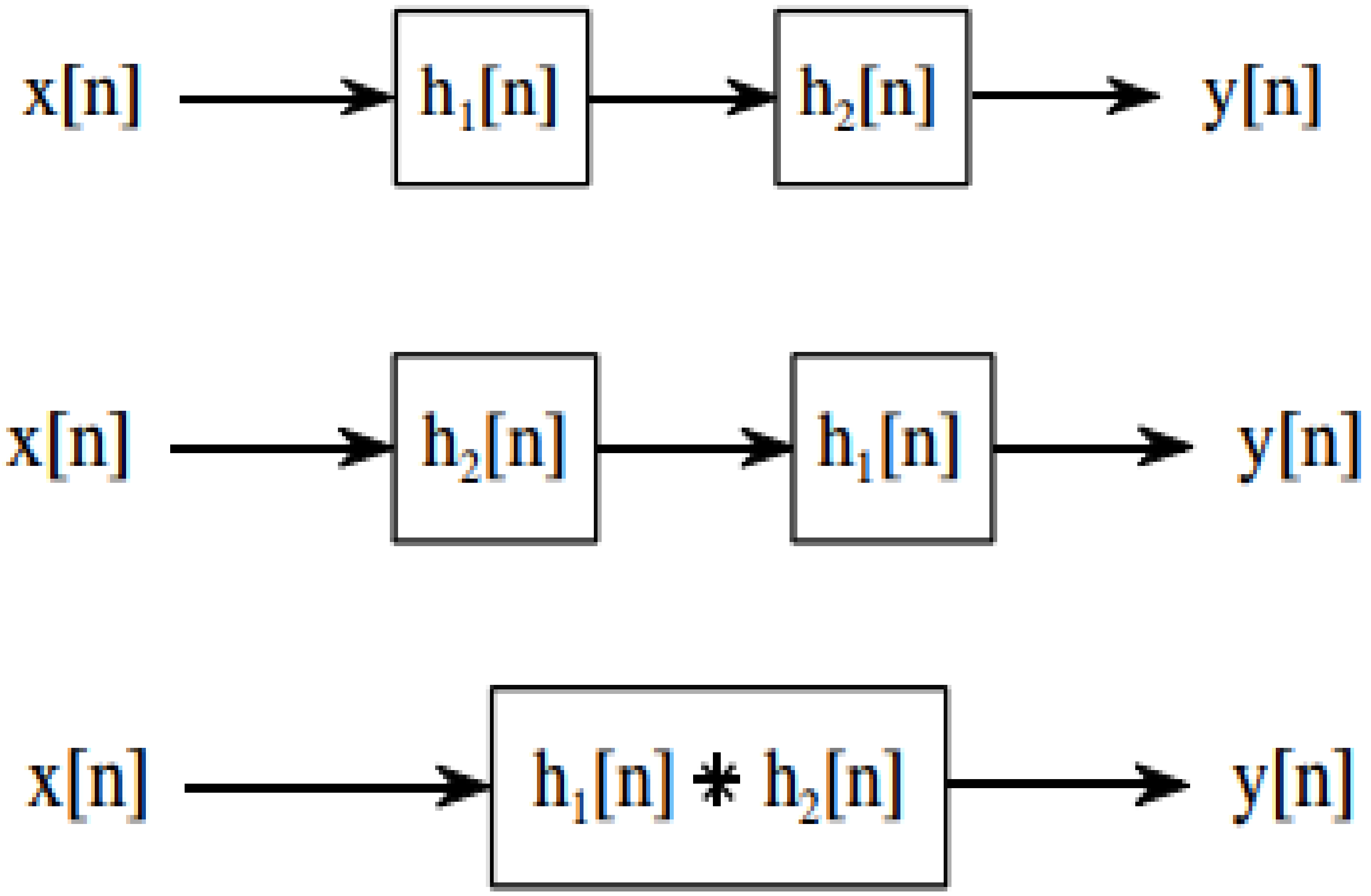
Commutative :

$$y(n) = x(n) * h(n) = h(n) * x(n)$$



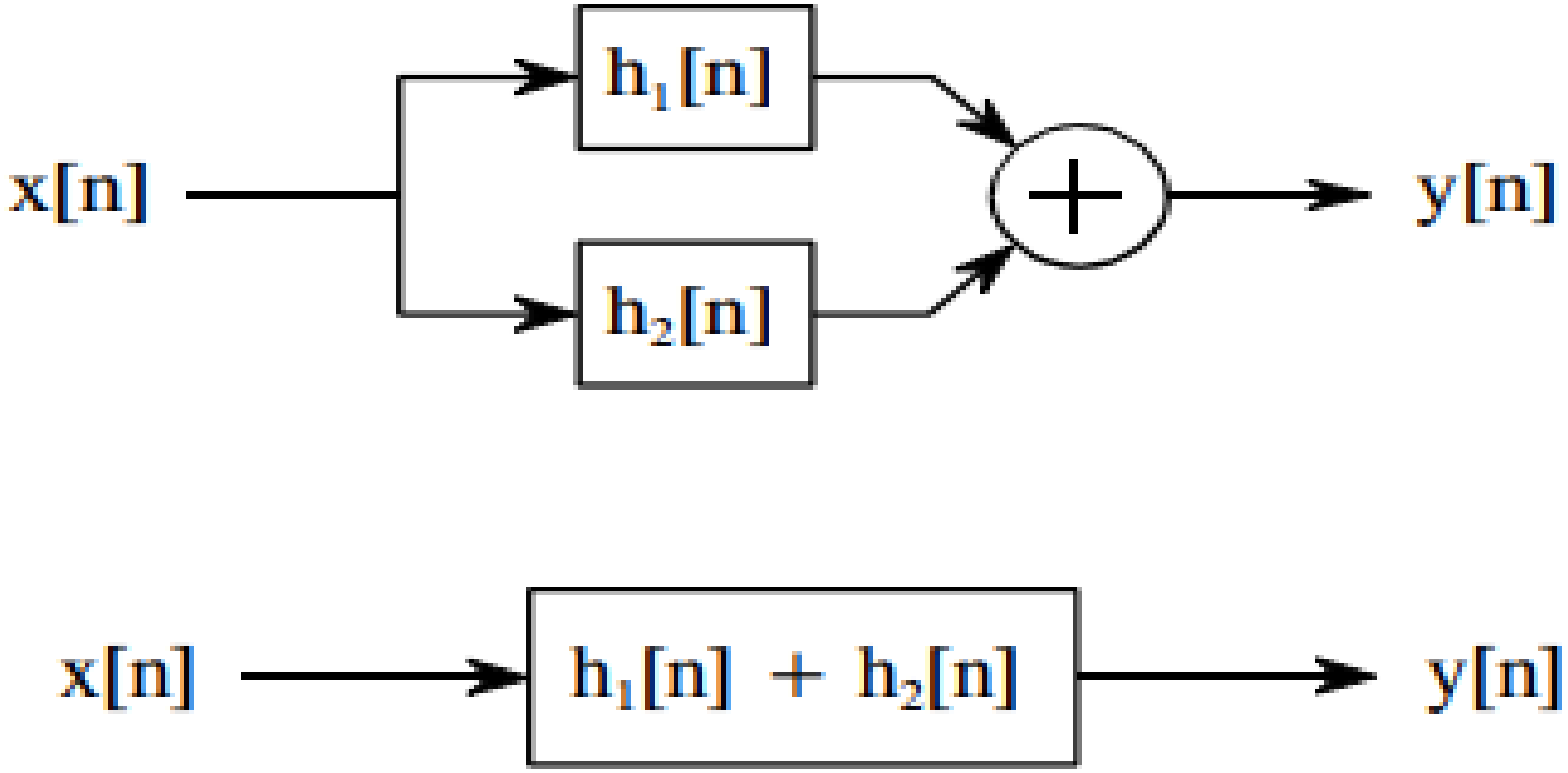
ASSOCIATIVE PROPERTY

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

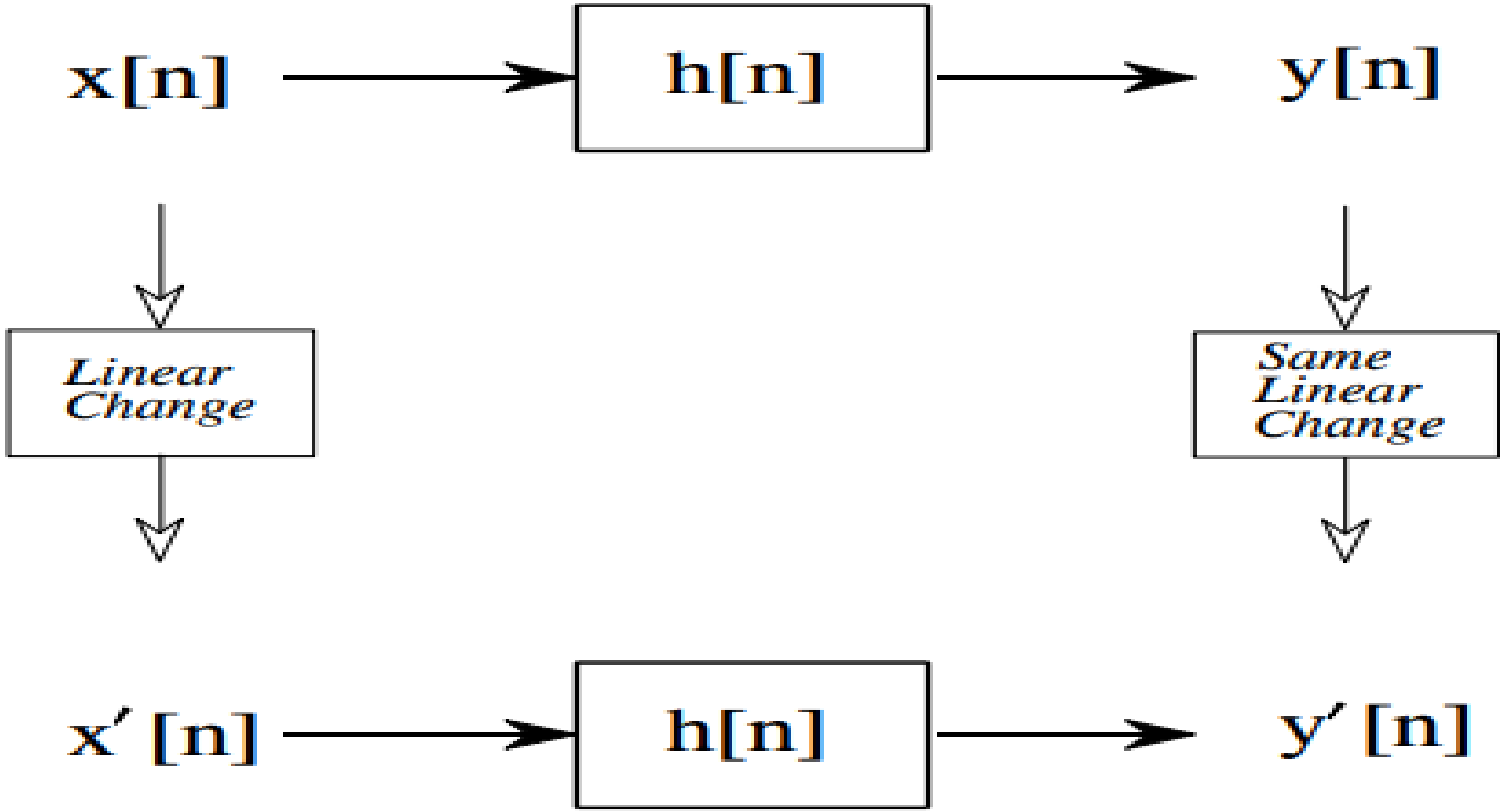


DISTRIBUTIVE PROPERTY

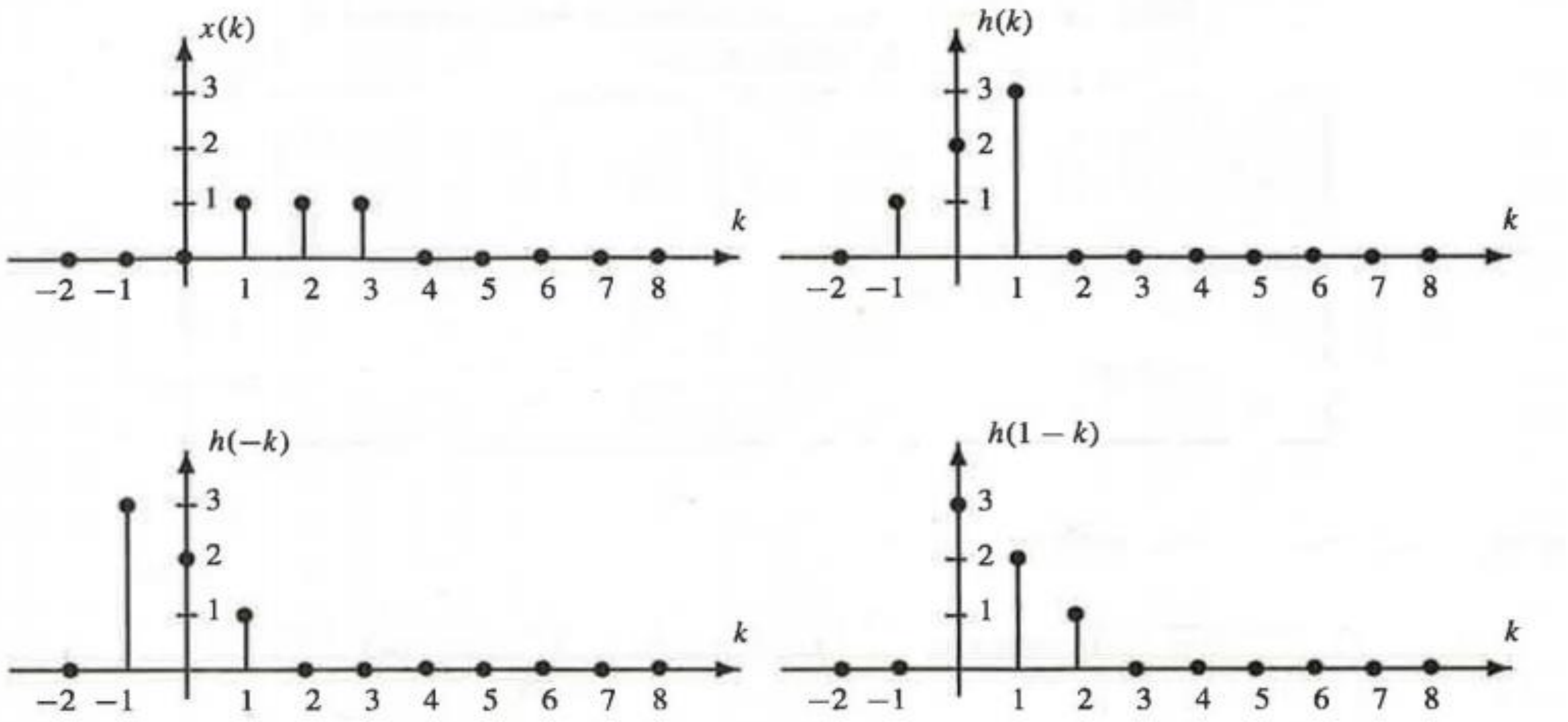
$$x(n) * h_1(n) + x(n) * h_2(n) = x(n) * [h_1(n) + h_2(n)]$$



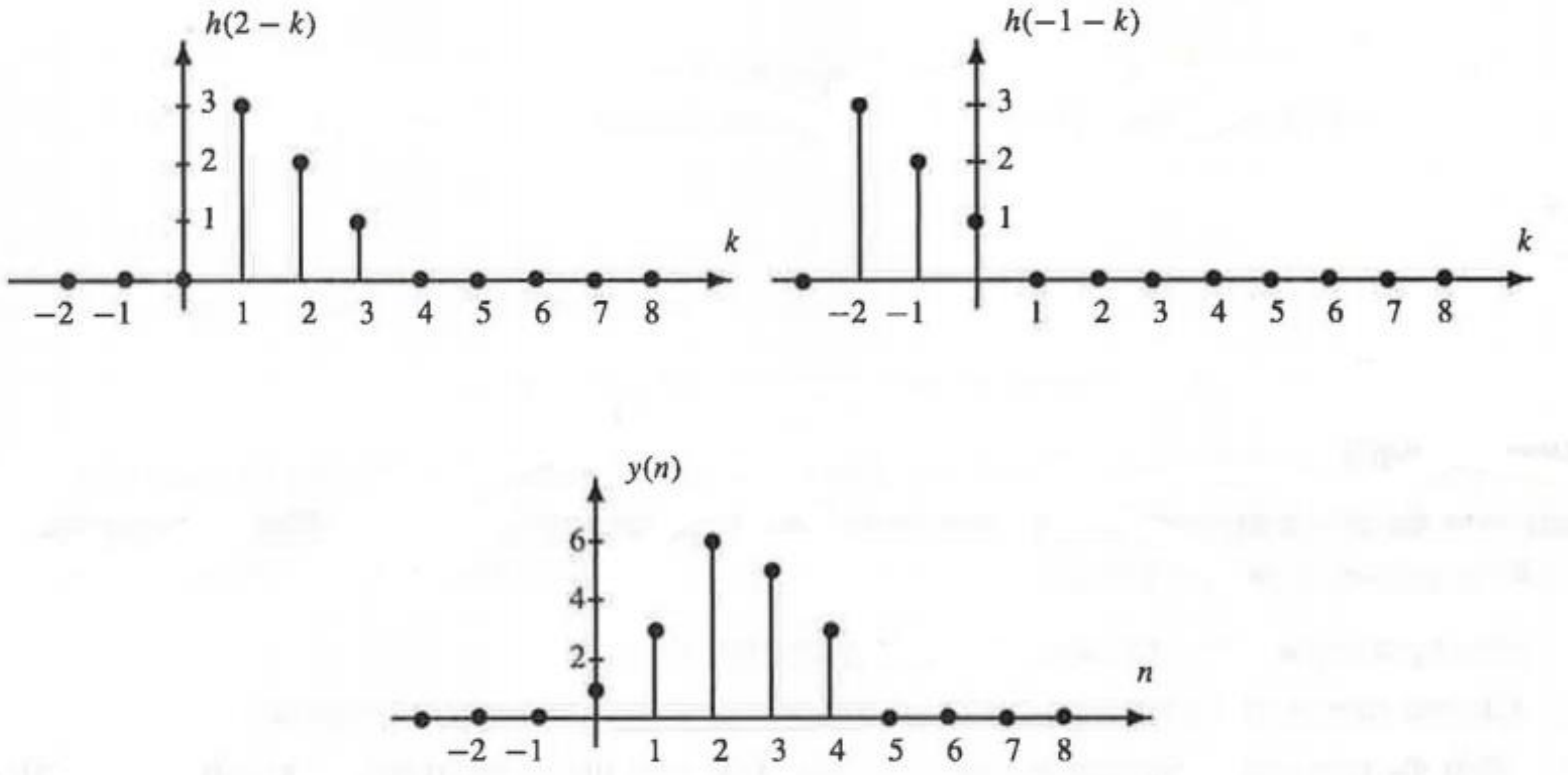
INPUT & OUTPUT TRANSFERENCE



GRAPHICAL REPRESENTATION



GRAPHICAL REPRESENTATION



MULTIPLICATION METHOD

Find the convolution sum of the sequence :-

$x(n) = \{1, 2, 3, 4, 5\}$ $h(n) = \{6, 7, 8\}$

		1	2	3	4	5
				6	7	8
		8	16	24	32	40
	7	14	21	28	35	
6	12	18	24	30		
6	19	40	61	82	67	40

$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$

TABULATION METHOD

$x(n) = \{1, 2, 3, 4, 5\}$ $\quad h(n) = \{6, 7, 8\}$

x(n) ↑ \ h(n) →	6	7	8
1	6	7	8
2	12	14	16
3	18	21	24
4	24	28	32
5	30	35	40

$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$

DEFINITION METHOD

$x(n) = \{1, 2, 3, 4, 5\}$ $\quad h(n) = \{6, 7, 8\}$

$x(0) = 1,\ x(1) = 2,\ x(2) = 3,\ x(3) = 4,\ x(4) = 5$

$h(0) = 6,\ h(1) = 7,\ h(2) = 8$

$\boxed{M = 5}$ $\quad \boxed{N = 3}$ $\quad y(n) = M + N - 1 \Rightarrow 5 + 3 - 1 \Rightarrow 7$

$y(n) = 7$ samples [n varies 0 to 6]

$$y(n) = \sum_{k=0}^{4} x(k)\, h(n-k)$$

$n = 0$

$$y(0) = \sum_{k=0}^{4} x(k)\, h(-k) \Rightarrow y(0) = 6$$

$n = 1$

$$y(1) = \sum_{k=0}^{4} x(k)\, h(1-k) \Rightarrow y(1) = 19$$

$n = 2$

$$y(2) = \sum_{k=0}^{4} x(k)\, h(2-k) \Rightarrow y(2) = 40$$

$n = 3$

$$y(3) = \sum_{k=0}^{4} x(k)\, h(3-k) \Rightarrow y(3) = 61$$

$n = 4$

$$y(4) = \sum_{k=0}^{4} x(k)\, h(4-k) \Rightarrow y(4) = 82$$

$n = 5$

$$y(5) = \sum_{k=0}^{4} x(k)\, h(5-k) \Rightarrow y(5) = 67$$

$n = 6$

$$y(6) = \sum_{k=0}^{4} x(k)\, h(6-k) \Rightarrow y(6) = 40$$

$$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$$

ASSESSMENT

1. Define convolution sum.
2. Total no. of samples in y(n) will be --------------
3. List the methods involved to compute convolution sum.
4. y(n) = x (n) * h(n) = h(n) * x(n) is defined as ------------- property
5. Mention the steps involved to compute linear convolution.
6. List the properties of convolution sum.

THANK YOU