Pipes and Cisterns

Problems on Pipes and Cisterns are based on the basic concept of time and work. Pipes are connected to a tank or cistern and are used to fill or empty the tank or cistern. In pipe and cistern, the work is done in form of filling or emptying a cistern/tank.

Inlet pipe : It fills a tank/cistern/reservoir.

Outlet pipe: It empties a tank/cistern/reservoir.

Important Points:

1. If a pipe can fill/empty a tank in 'm' h, then the part of tank filled /emptied in $1\ h = \frac{1}{m}$.

For example, A pipe can fill the tank in 7 h, then the volume of tank filled in $1\ h = \frac{1}{7}$.

2. If a pipe can fill/empty '$\frac{1}{m}$' part of a tank in 1 h, then it can fill/empty the whole tank in $'m'$ h.

For example, If a pipe can fill $\frac{1}{5}$ part of a tank in 1 h, then it can fill the whole tank in 5 h.

3. Generally, time taken to fill a tank is taken positive (+ ve) and time taken to empty a tank is taken negative (- ve).
4. If a pipe fills a tank in m h and an another pipe fills in n h. Then, part filled by both pipes in $1\ h = \frac{1}{m} + \frac{1}{n}$.

Example : 1 An outlet pipe can empty a cistern in 5 h. In what time will the pipe empty $\frac{2}{5}$ part of the cistern?
Solution : Time taken to empty full cistern = 5 h

Time taken to empty $\frac{2}{5}$ part of the cistern = $\frac{2}{5} \times 5 = 2\ h$

Example : 2 If a pipe can fill a tank in 2 h and another pipe can fill the same tank in 6 h, then what part of a tank will be filled by both the pipes in 1 h, if they are opened simultaneously?

Solution : In 1 h, part filled by 1st pipe = $\frac{1}{m} = \frac{1}{2}$

In 1 h, part filled by 2nd pipe = $\frac{1}{n} = \frac{1}{6}$.

In 1 h, part filled by both the pipes together = $\left(\frac{1}{m} + \frac{1}{n}\right) = \left(\frac{1}{2} + \frac{1}{6}\right) = \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3}$ part

Example : 3 If a pipe can fill a tank in 5 h and an another pipe can empty the tank in 10 h, then part fill by both pipes in 1 h, if both pipes are open simultaneously.

Solution: In 1 h, part filled by 1st pipe = $\frac{1}{m} = \frac{1}{5}$

In 1 h, part emptied by 2nd pipe = $\frac{1}{n} = \frac{1}{10}$

In 1 h, part filled by both pipes when open simultaneously = $\frac{1}{m} - \frac{1}{n}$

$= \frac{1}{5} - \frac{1}{10} = \frac{2-1}{10} = \frac{1}{10}$ part [-ve sign is used, as 2nd pipe empties the tank]

Fast Track Techniques to solve the Questions

Technique :1

If a pipe can fill/empty a tank in $'m'$ h and an another pipe can fill/empty the same tank in $'n'$ h, then

(i) If both pipes either fills or empties the tank, then the time taken to fill or empty the tank when both pipes are opened is $t = \frac{mn}{m+n}$

(ii) If first pipe fills the tank and second pipe empties the tank, then the time taken to fill the tank when both pipes are opened is $t = \frac{mn}{m-n} : m > n$

(iii) If first pipe fills the tank and second pipe empties the tank, then the time taken to empty the tank when both pipes are opened is $t = \frac{mn}{n-m} : n > m$

Example : 4 Two pipes A and B can fill a tank in 18 h and 12 h, respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Solution: Time taken by both pipes to fill the tank= $\frac{mn}{m+n}$ where m and n are the time taken to fill the tank by individual pipes.

Here, $m = 18, n = 12$

Time taken to fill the tank $= \frac{m \times n}{m+n} = \frac{18 \times 12}{18+12} = \frac{18 \times 12}{30} = \frac{3 \times 12}{5} = \frac{36}{5} = 7\frac{1}{5}\ h$

Example : 5 A pipe can fill a tank in 5 h, while another pipe can empty it in 6 h. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Solution : Here, m= 5 h and n = 6 h

Time taken to fill the tank = $\frac{m \times n}{n-m} = \frac{5 \times 6}{6-5} = \frac{30}{1} = 30\ h$

Example : 6 A pipe can fill a tank in 10 h. Due to a leak in the bottom, it fills the tank in 20 h. If the tank is full, how much time will the leak take to empty it?

Solution: Here, $m = 10$ and $n = 20$

According to the formula,

Required time taken to empty the tank = $\frac{m\times n}{n-m} = \frac{10\times 20}{20-10} = \frac{200}{10} = 20\ h$

Technique :2

If three pipes can fill a tank separately in m,n and p h, respectively, then part of tank filled in 1 h by all the three pipes is given by $\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{p}\right)$ and total time taken to fill the tank is given by $\frac{mnp}{np+mp+mn}h$.

Note : If any one of the three pipes is used to empty the tank, then time taken by that particular pipe will be negative (-ve). Suppose, 3rd pipe is used to empty the tank. Then, the above formulae takes the form as $\left(\frac{1}{m}+\frac{1}{n}-\frac{1}{p}\right)$ and $\frac{mnp}{np+mp-mn}h$.

Example : 7 Three pipes m, n and p can fill a tank separately in 4,5 and 10 h, respectively. Find the time taken by all the three pipes to fill the tank when the pipes are opened together.

Solution : Part filled by pipe m in 1 h = $\frac{1}{4}$

Part filled by pipe n in 1 h = $\frac{1}{5}$

Part filled by $(m+n+p)$ pipes in 1 h = $\frac{1}{4}+\frac{1}{5}+\frac{1}{10} = \frac{5+4+2}{20} = \frac{11}{20}$

Required time to fill the tank = $\frac{20}{11}h = 1\frac{9}{11}h$.

Example : 8 Pipe A can fill a tank in 20 h while pipe B alone can fill it in 10 h and pipe C can empty the full tank in 30 h. If all the pipes are opened together, how much time will be needed to male the tank full?

Solution : Here, $m = 20, n = 10$ and p=30

Required time to fill the tank = $\frac{mnp}{np+mp-mn} = \frac{20\times 10\times 30}{10\times 30+20\times 30-20\times 10}$

$$= \frac{6000}{300+600-200} = \frac{6000}{700} = \frac{60}{7} = 8\frac{4}{7}h.$$

Technique : 3

Two pipes A and B together can fill a tank in time t. If time taken by A alone is more than t by a and time taken by B alone is more than t by b, then $t = \sqrt{ab}$.

Example : 9 Two pipes A and B are opened together to fill a tank. Both the pipes fill the tank in time t. If A separately takes 4 min more time than t to fill the tank and B takes 64 min more than t to fill the tank, find the value of t.

Solution : We know that, time taken by both pipes to fill the tank (t) = $\sqrt{ab}$

Here a=4 and b= 64

$t = \sqrt{4\times 64} = 2\times 8 = 16$ min

Technique : 4

A full tank get emptied in $'a'$ h due to presence of a leak in it. If a tap which fills it at a rate of $'b'\ L/h$, is opened, then it get emptied in $'c'$ h.

Therefore, volume of tank = $\frac{abc}{c-a}$

Example : 10 A full tank get emptied in 6 min due to presence of an orifice in it. On opening a tap which can fill the tank at the rate of $8\ L/min$, the tank get emptied in 10 min. Find the capacity of tank.

Solution: Here, $a = 6, b = 8$ and $c = 10$

Capacity of tank $= \frac{abc}{c-a} = \frac{6\times8\times10}{10-6} = 120\ L.$

Technique : 5

If two taps A and B, which can fill a tank, such that efficieny of A is n times of B and takes t min less/more than B to fill the tank, then

(i) Time taken to fill the tank by both pipes together = $\frac{nt}{n^2-1}\ min$

(ii) Time taken to fill the tank by faster tap = $\frac{t}{n-1}\ min$

(iii) Time taken to fill the tank by slower tap = $\frac{nt}{n-1}\ min$

Example : 11 If tap A can fill a tank 3 times faster than tap B and takes 28 min less than tap B to fill the tank. If both the taps are opened simultaneously, then find the time taken to fill the tank.

Solution: Here, n = 3 and t = 28

According to the formula,

So, time taken to fill the tank by both pipes together = $\frac{nt}{n^2-1} = \frac{28\times3}{(3)^2-1} = \frac{21}{2}\ min$

Technique : 6

Two pipes A and B can fill a tank in x min and y min, respectively. If both the pipes are opened simultaneously, then the time after which pipe B should be closed so that the tank is full in t min, is $\left[y\left(1-\frac{t}{x}\right)\right]\ min.$

Example : 12 Two pipes A and B can fill a tank in 12 and 16 min, respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 9 min?

Solution: Here, $x = 12, y = 16$ and $t = 9$

Required time after which B should be closed = $y\left(1-\frac{t}{x}\right) = 16\left(1-\frac{9}{12}\right) = 16\times\frac{3}{2} = 4\ min.$