23GET276 - VQAR-II

UNIT I - QUANTITATIVE ABILITY III

Pipes & Cisterns

INLET: An inlet is a pipe which is connected to the tank and with the help of this pipe, the tank is filled.

OUTLET/LEAK: An outlet is a pipe which is connected to the tank. This pipe drains out water from the tank and the tank gets emptied if this pipe is opened.

Formulae

1. If a pipe can fill a tank in a hrs, then the part filled in 1 hr =1/a.

2. If a pipe can empty a tank in b hrs, then the part of the full tank emptied in 1 hr = 1/b.

3. If a pipe can fill a tank in a hrs and the another pipe can empty the full tank in b hrs, then the net part filled in 1 hr, when both the pipes are opened =[1/a - 1/b]
   ∴ Time taken to fill the tank, when both the pipes are opened = ab/(b - a)

4. If a pipe can fill a tank in a hrs and another can fill the same tank in b hrs, then the net part filled in 1 hr, when both pipes are opened = [1/a + 1/b]
   ∴ Time taken to fill the tank = ab/(a + b)

5. If a pipe fills a tank in a hrs and another fills the same tank in b hrs, but a third one empties the full tank in c hrs, and all of them are opened together, the net part filled in 1 hr = [1/a + 1/b - 1/c]

∴ Time taken to fill the tank =abc/(bc + ac – ab) hrs.

6. A pipe can fill a tank in a hrs. Due to a leak in the bottom it is filled in b hrs. If the tank is full, the time taken by the leak to empty the tank = ab/(b - a) hrs

Example 1: If a pipe fills a tank in 20 minutes and a pipe empties the same tank in 60 minutes. Then in how much time the tank will be filled completely if both the pipes are opened together?

Sol: Let the tank be filled in X minutes.
By unitary method,
(1/20) - (1/60) = (1/X).
Solving the equation, we get, x = 30 minutes. So, the tank will be filled in 30 minutes.

Example 2: Two pipes fill a tank in 20 minutes and 60 minutes. A third pipe empties the same tank in 40 minutes. In how much time will the tank be filled if all three pipes are opened together?

Sol: Let the tank is filled in X minutes. By unitary method,
(1/20) + (1/60) - (1/40) = (1/X).
Solving, we get x = 24 minutes. Hence, the tank will be filled in 24 minutes.

Example 3: A pipe can fill a cistern in 12 minutes and another can fill it in 15 minutes, but a third pipe can empty it in 6 minutes. The first two are kept open for 6 minutes in the beginning and then the third pipe is also opened, in what time will the cistern be emptied?

Solution:

Let the capacity of the tank = LCM of (12,15, 6) = 60 litres.

Rate of work done by the first pipe = 60/12 ⇒ 5 L/min

Rate of work done by the second pipe = 60/15 ⇒ 4L/min

Rate of work done by the draining pipe = 60/6 ⇒ 10L/min

If all the pipes are opened simultaneously, (5+4-10) litres will be filled in a minute.

Since this comes out to be negative, no water will be filled at any point of time.

For the 6 minutes only filling pipes are opened,

Rate of both filling pipes = 9L/min, (5+4).

So in 6 minutes, 9 × 6 = 54 litres water is filled.

When the third pipe is opened, all three pipes will work simultaneously,

at a rate of (5+4 – 10) = - 1 ltr per min. This implies that in a minute, 1 litre water will be drained.

So, 54 litres water will be drained in 54 minutes. So the tank will be emptied in 54 minutes.

Example 4: Two pipes M and N can fill a tank in 20 hours and 30 hours respectively. If both pipes are opened together, then how much time it will take to fill the tank?

Sol:

Part of the tank filled by M in one hour = 1/20

Part of the tank filled by N in one hour = 1/30

Hence part filled by (M + N) in I hour = 1/20 + 1/30 = 1/12

Hence both pipes can fill the tank together in 12 hours

OR direct formula can also be used as (20 × 30)/20 + 30 = 600/50 = 12 hours

Example 5: A pipe can fill a tank in 40 hours. Due to a leak in the bottom, the tank is getting filled in 60 hours. In how much time the tank would be empty if leak is working alone?

Sol: Let us assume that leak empties the tank in x hours.

So 1/40 – 1/x = 1/60.

Solving this equation, we get x = 120. So the tank would be empty in 120 hours.

Example 6: Two pipes A and B fill a tank in 30 minutes and 60 minutes respectively. A pipe C at the bottom can empty the tank in 120 minutes. If all three pipes were open simultaneously, how long does it take to fill the empty tank?

Sol: Pipe A fills 1/30th of the tank in a minute

Pipe B fills 1/60th of the tank in a minute

Pipe C drains 1/120th of the tank in a minute.

Therefore, if all three are open then, (1/30 + 1/60 – 1/120)th of the tank will be filled in a min.

i.e. 1/24 th of the tank will be filled in a minute.

Therefore, the tank will be filled in 24 minutes.

Example 7: Two filling pipes M and N can fill a tank in 20 hours and 60 hours respectively. There is an outlet P also. If all the pipes are opened together, then tank is full in 40 hours. How much time would be taken by P to empty the full tank if working alone?

Sol: Let us assume that leak empties the tank in x hours.

So 1/20 + 1/60 – 1/x = 1/40.

Solving this equation, we get x = 24.

Hence the tank would be empty in 24 hours.

Example 8: Pipes M and N can fill a tank in 60 minutes and 30 minutes respectively. Pipe P drains 24 litres of water in a minute. If all of them are kept open when the tank is full, the tank gets emptied in 60 minutes. How much water can the tank hold?

Sol. Let the drain pipe takes E minutes to empty the tank.

Now 1/60 + 1/30 – 1/E = -1/60.

The value on the Right Hand Side would be negative because full tank has been emptied.

Solving this we get E = 15.

If the drain pipe can empty a tank in 15 minutes and its drainage rate is 24 litres per minute, the capacity will be 15 x 24 = 360 litres.

Example 9: Three pipes A, B and C can fill a cistern in 12 hours. After working at it for 6 hours, C is closed and A and B can fill it in 10 hours more. How many hours will C alone take to fill the cistern?

Sol. In 6 hours A, B and C would have filled 6 x 1/12 = 1/2 of the cistern.

Remaining part is 1 – 1/2 = 1/2, which A and B have filled in 10 hours.

→ A and B can fill the whole cistern in 10 x 2/1 = 20 hours.

Its given that A, B and C can fill the tank in 12 hours.

So C can fill the cistern alone in 1/12 – 1/20 = 1/30 = 30.

Hence C alone would take 30 hours to fill the tank.

Example 10: If two pipes work simultaneously, the tank gets filled in 24 hours. One pipe takes 20 hours longer than the other. How many hours does faster pipe takes to fill the tank working alone?

Sol. Let us assume that faster pipe takes x hours to fill the tank working alone.

Then the slower pipe takes (x + 20) hours to fill the tank working alone.

When both of them are working together, it takes 24 hours to fill the tank.

Hence $1/x + 1/(x +20) = 1/24$.

Solving this equation, we get $x = 40$.

So the faster pipe takes 40 hours to fill the tank working alone.