



DEPARTMENT OF MATHEMATICS

UNIT - II DESIGN OF EXPERIMENTS

LATIN SQUARE :

2) An agriculturist wants to test the effects of four different fertilizers A, B, C and D on the yield of paddy. In order to eliminate sources of error due to variability in self-fertility he used the fertilizers in a Latin square arrangement as given below where the numbers indicate yields in quintals per unit area. Perform an analysis of variance to decide whether there is a difference between the fertilizers at 5% level of significance.

A	18	D	20	C	23	B	11
D	22	A	20	B	10	C	19
B	15	C	21	D	25	A	17
C	22	B	12	A	15	D	24

Soln: Let origin = $n_{ij} - 18$ avg(min, max)

	x_1	x_2	x_3	x_4	total	x_1^2	x_2^2	x_3^2	x_4^2
y_1	0	3	5	-7	1	0	9	25	49
y_2	4	+2	-8	1	-1	16	4	64	1
y_3	-3	3	7	-1	6	9	9	49	1
y_4	4	-6	-3	6	1	16	36	9	36
total	5	2	1	-1	7	41	58	147	87
	$\sum x_1$	$\sum x_2$	$\sum x_3$	$\sum x_4$		$\sum x_1^2$	$\sum x_2^2$	$\sum x_3^2$	$\sum x_4^2$

Step 1: Formulate H_0 & H_1 :

H_0 : There is no difference between the fertilizers

H_1 : There is difference between the fertilizers.



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step 2: To find T & N :

$$T = \sum n_1 + \sum n_2 + \sum n_3 + \sum n_4$$

$$= 5 + 2 + 1 + 1 = 9$$

$$N = n_1 + n_2 + n_3 + n_4$$

$$= 4 + 4 + 4 + 4 = 16$$

step 3: To find correction factor, C.F.

$$C.F. = \frac{T^2}{N} = \frac{9^2}{16} = \frac{81}{16} = 5.0625$$

step 4: To find TSS;

$$TSS = \sum n_1^2 + \sum n_2^2 + \sum n_3^2 + \sum n_4^2 - C.F.$$

$$= 41 + 58 + 14 + 8 - 5.0625$$

$$= 121 - 5.0625 = 115.9375$$

step 5: To find SSC, SSR, & SST

$$SSC = \frac{(\sum n_1)^2}{n_1} + \frac{(\sum n_2)^2}{n_2} + \frac{(\sum n_3)^2}{n_3} + \frac{(\sum n_4)^2}{n_4} - C.F.$$

$$= \frac{5^2}{4} + \frac{2^2}{4} + \frac{1^2}{4} + \frac{1^2}{4} - 5.0625$$

$$= 4.6875$$

$$SSR = \frac{(\sum y_1)^2}{n_1'} + \frac{(\sum y_2)^2}{n_2'} + \frac{(\sum y_3)^2}{n_3'} + \frac{(\sum y_4)^2}{n_4'} - C.F.$$

$$= \frac{1^2}{4} + \frac{1^2}{4} + \frac{6^2}{4} + \frac{1^2}{4} - 5.0625$$

$$= 6.6875$$



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To find SST :

A	0	2	-3	-1	-2 : Σz_1
B	-3	-6	-8	-7	-24 : Σz_2
C	4	3	5	1	13 : Σz_3
D	4	3	7	6	20 : Σz_4

$$\begin{aligned} SST &= \frac{(\Sigma z_1)^2}{4} + \frac{(\Sigma z_2)^2}{4} + \frac{(\Sigma z_3)^2}{4} + \frac{(\Sigma z_4)^2}{4} - C.F. \\ &= \frac{-2^2}{4} + \frac{-24^2}{4} + \frac{13^2}{4} + \frac{20^2}{4} - C.F. \\ &= 284.25 - 3.0625 = 284.1875 \end{aligned}$$

step 6: to find SSE

$$\begin{aligned} SSE &= TSS - SSSC - SSR - SST \\ &= 329.94 - 4.6875 - 6.6875 - 284.1875 \\ &= 34.375 \end{aligned}$$



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Step 7: Anova table.

Source of variations	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	F-Ratio
Column	SSC: 4.6875	$c-1 = 3$	$MSC = \frac{4.6875}{3}$ $= 1.5625$	$F_c = \frac{5.7291}{1.5625}$ $= 3.6666$ $F_{\alpha}(6, 3) = 9.94$
Row	SSR: 6.6875	$r-1 = 3$	$MSR = \frac{6.6875}{3}$ $= 2.2291$	$F_r = \frac{5.7291}{2.2291}$ $= 2.5701$ $F_{\alpha}(6, 3) = 9.94$
Treatment	SST: 284.1875	$T-1 = 3$	$MST = \frac{284.1875}{3}$ $= 94.7291$	$F_T = \frac{94.7291}{16.5347}$ $= 5.7291$ $F_{\alpha}(3, 6) = 4.76$
Error	SSE: 34.375	$(n-1)(n-2)$ $3 \times 2 = 6$	$MSE = \frac{34.375}{6}$ $= 5.7291$	

Step 8: Conclusion:

$$F_c = 3.6666 < 9.94 = F_{\alpha}, \quad H_0 \text{ is accepted}$$

$$F_r = 2.5701 < 9.94 = F_{\alpha}, \quad H_0 \text{ is accepted}$$

$$F_T = 16.5347 > 4.76 = F_{\alpha}, \quad H_0 \text{ is rejected}$$

(a) There is difference between the fertilizers.



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2) Analyse the variance in the Latin square of yields (in quintals) of wheat where P, Q, R, S represent the different manures used.

S 222 P 221 R 223 Q 222

Q 224 R 223 P 222 S 225

P 220 Q 219 S 220 R 221

R 222 S 223 Q 221 P 222

Test whether the different manures used have given significantly different yields:

Soln: $F_c = 1.34$; $F_R = 12.31$, $F_T = 2.12$ & $F_x = 4.76$