

# SNS COLLEGE OF TECHNOLOGY



(An Autonomous Institution) Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & amp; Accredited by NBA (B.E - CSE, EEE, ECE, Mech & amp; B.Tech.IT) COIMBATORE-641 035, TAMIL NADU

VECTOR CALCULUS Scalar Quantity : A scalar quantity is that which has magnitude and is not related to any direction. Vector Quantity: A vector vuantity is that which has both magnitude and direction. Scalar Point function : 10000 12 placounitars at hom If corresponding to each point P of a sugion R there corresponds a scalar denoted by Q(P) or p(x,y,z) then p is said to be a scalar point function for the stegion R. Example: The temperature Q(P) at any point P of a body occupying a certain sugion is a scalar point function . Vector point function : If corresponding to each point P of a Stegion R, there corresponds a vector denoted by F(P), then F is said to be a vector point function for the Region R. Example: The acceleration F(P) of a particle at any time to occupying the position P in a certain region is a vector point function.  $brook (f \pm g) = qrad (f \pm g)$ 



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Vector Differential Operator: The vector differential operator & is defined as,  $\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} = \vec{z} \vec{i} \frac{\partial}{\partial x}$ Gradient of a scalar point function : Let  $\varphi(x, y, z)$  be a scalar point function and is continuously differentiable then the vector  $\nabla \varphi = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}\right) \varphi \Big|$  $= \vec{i} \frac{\partial \varphi}{\partial x} + \vec{j} \frac{\partial \varphi}{\partial y} + \vec{k} \frac{\partial \varphi}{\partial z}$ is called the gradient of  $\varphi$  and is written as grad  $\varphi$ . i.e., grad  $\varphi = \nabla \varphi$ Note : 1. Vo defines a vector field. 2.  $\nabla \phi \neq \phi \nabla$ . There will be no '.' or 'x'. between  $\varphi$  and  $\nabla$ . and  $\varphi$  himself is a set Properties of Gradient: 1. If f and g are two scalar point functions then.  $\nabla(f \pm g) = \nabla f \pm \nabla g$ (or)  $grad (f \pm g) = grad f \pm grad g$ 





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(2) If f and g are two scalar point functions then,  

$$\nabla (fg) = f \forall g + g \forall F$$
(or)  $grad (fg) = f(grad g) + g(grad f)$ 
(3) If f and g are two scalar point functions then,  
 $\left[ \forall \left(\frac{f}{g}\right) = \frac{g \forall F - f \forall g}{g^2} \text{ where } g \neq o \right]$   
 $grad \left(\frac{f}{g}\right) = \frac{g(grad f) - f(grad g)}{g^2}$ 
(c)  $grad \left(\frac{f}{g}\right) = \frac{g(grad f) - f(grad g)}{g^2}$ 
(c)  $Gradient of a constant is zero
 $i.e., \forall \nabla \phi = 0$   
Problems:  
(1) Find grad  $\phi$  where  $\phi = x^2 + y^2 + z^2$   
Soln:  
 $\forall \phi = \vec{i} \cdot \frac{\partial \phi}{\partial x} + \vec{j} \cdot \frac{\partial \phi}{\partial y} + \vec{k} \cdot \frac{\partial \phi}{\partial z}$   
 $= \vec{i} \cdot \frac{\partial}{\partial x} (x^2 + y^2 + z^2) + \vec{j} \cdot \frac{\partial}{\partial y} (x^2 + y^2 + z^2) + \vec{k} \cdot \frac{\partial}{\partial z} (x^2 + y^2 + z^2)$   
 $= \vec{i} \cdot (2x) + \vec{j} \cdot (2y) + \vec{k} \cdot (2x)$   
 $\forall \phi = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$   
(2) Find grad  $\phi$  if  $\phi = xyz$  at  $(1, 1, 1)$   
 $\leq oln:$   
 $\forall \phi = \vec{i} \cdot \frac{\partial \phi}{\partial x} + \vec{j} \cdot \frac{\partial \phi}{\partial y} + \vec{k} \cdot \frac{\partial \phi}{\partial z}$$ 



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 $\nabla \varphi = \vec{i} \frac{\partial}{\partial x} (xyz) + \vec{j} \frac{\partial}{\partial y} (xyz) + \vec{k} \frac{\partial}{\partial z} (xyz)$  $= \vec{i} yz + \vec{j} (xz) + \vec{k} (xy)$  $\nabla \varphi_{(1,1,1)} = \vec{i} + \vec{j} + \vec{k}.$ Find grad  $\rho$  where  $\phi = 3x^2y - y^3z^2$  at (1,1,1).  $\nabla \varphi = \vec{i} \frac{\partial \varphi}{\partial x} + \vec{j} \frac{\partial \varphi}{\partial y} + \vec{k} \frac{\partial \varphi}{\partial z}$  $=\overline{i}\frac{\partial}{\partial x}(3x^{2}y-y^{3}z^{2})+\overline{j}\frac{\partial}{\partial y}(3x^{2}y-y^{3}z^{2})$  $+ \frac{\partial}{\partial z} \left( 3x^{2}y - y^{3}z^{2} \right)$  $= \vec{i} (6xy) + \vec{j} \cdot (3x^2 - 3y^2 z^2) + \vec{k} (-2y^3 z)$  $\nabla \varphi_{(1,1,1)} = 6\vec{i} - 2\vec{k}$ If  $\varphi = \log(2x^2 + y^2 + z^2)$  find  $\nabla \varphi$  borg borg () soln:  $\nabla \varphi = \vec{i} \frac{\partial \varphi}{\partial x} + \vec{j} \frac{\partial \varphi}{\partial u} + \vec{k} \frac{\partial \varphi}{\partial v}$  $= \overline{i} \frac{\partial}{\partial x} \left[ \log \left( x^2 + y^2 + z^2 \right) \right] + \overline{j} \frac{\partial}{\partial y} \left[ \log \left( x^2 + y^2 + z^2 \right) \right]$  $+ \vec{k} \frac{\partial}{\partial z} \left[ \log \left( x^2 + y^2 + z^2 \right) \right]$  $= \frac{1}{1} \frac{-1}{x^2 + y^2 + z^3} (2x) + \frac{1}{y} \frac{1}{x^2 + y^3 + z^3} (2y) + \frac{1}{x^2 + y^3 + z^3} (2$  $(x_1, y_2, y_3)$  to  $x_2 \times \overline{K^2 + y_2^2} = (2z) p$  and (g) $= \frac{a}{n^2 + y^2 + z^2} (n\vec{i} + y\vec{j} + z\vec{k})$