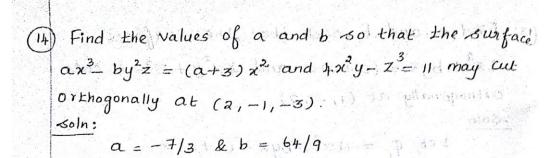


SIS

(An Autonomous Institution)

DEPARTMENT OF MATHEMATICS



DIVERGENCE OF A VECTOR POINT FUNCTION:

Let F be any given continuously differentiable vector point function then the divergence of F is defined as,

$$\operatorname{div} \vec{F} = \nabla \cdot \vec{F} = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}\right) \cdot \vec{F}$$

$$= \vec{i} \cdot \frac{\partial \vec{F}}{\partial x} + \vec{j} \cdot \frac{\partial \vec{F}}{\partial y} + \vec{k} \cdot \frac{\partial \vec{F}}{\partial z}$$

Note :

1. V. F is a scalar point function.

Solenoidal vector: 0 = . 9V .

Vector F is said to be solenoidal vector if div F = 0.

CURL OF A VECTOR POINT FUNCTION:

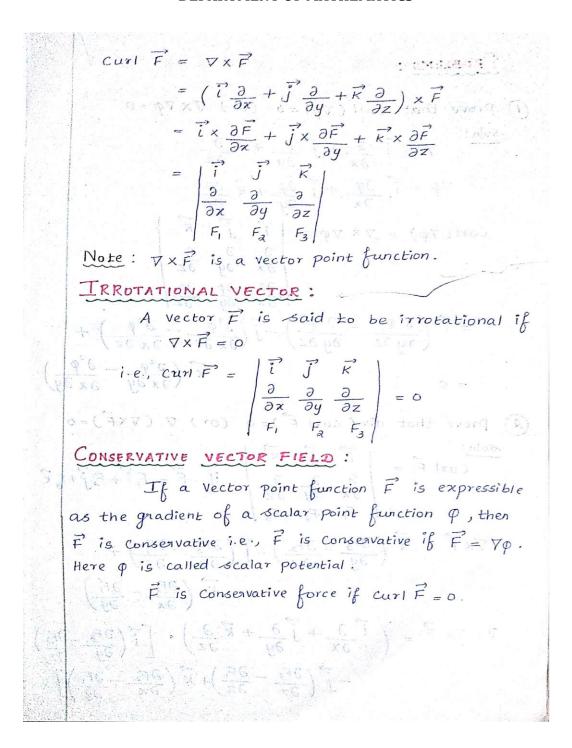
Let $\vec{F} = F_1 \vec{i} + F_2 \vec{j} + F_3 \vec{k}$ be any given continuously differentiable vector point function, the curl or rotation of \vec{F} is defined as,





(An Autonomous Institution)

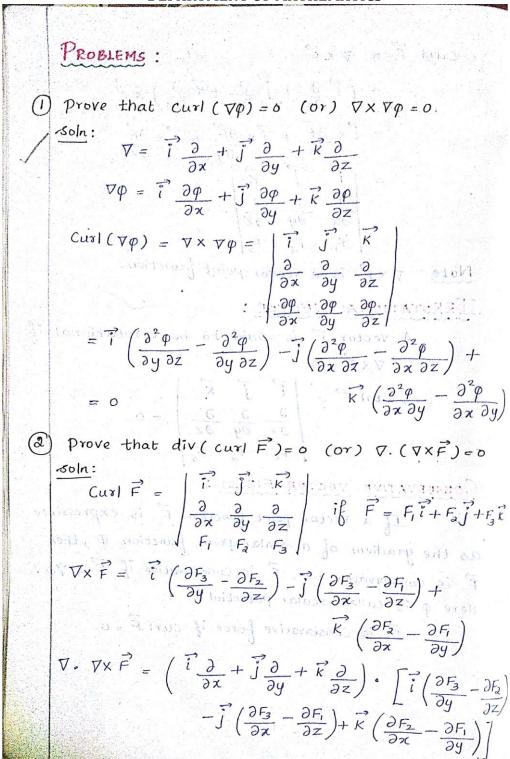
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & Colombia (B.E - CSE, EEE, ECE, Mech & Colombia (B.E - CSE), TAMIL NADU







(An Autonomous Institution)







(An Autonomous Institution)

Find the constants
$$a, b, c > 0$$
 that the Vector $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.

Soln:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix}
3/\partial x & 3/\partial y & 3/\partial z \\
x + 2y + az & bx - 3y - z & 4x + cy + az
\end{vmatrix}$$

$$= \vec{i} \begin{vmatrix}
3/\partial x & (4x + cy + az) - \frac{\partial}{\partial z} & (bx - 3y - z)
\end{vmatrix}$$

$$-\vec{j} \begin{vmatrix}
3/\partial x & (4x + cy + az) - \frac{\partial}{\partial z} & (x + 2y + az)
\end{vmatrix}$$

$$+ \vec{k} \begin{vmatrix}
3/\partial x & (bx - 3y - z) - \frac{\partial}{\partial z} & (x + 2y + az)
\end{vmatrix}$$

$$+ \vec{k} \begin{vmatrix}
3/\partial x & (bx - 3y - z) - \frac{\partial}{\partial z} & (x + 2y + az)
\end{vmatrix}$$

$$= \vec{i} (c+1) - \vec{j} (4-a) + \vec{k} (b-2)$$
Given: \vec{F} is irrotational.

i.e., $\nabla \times \vec{F} = 0$.

$$\vec{i} (c+1) - \vec{j} (4-a) + \vec{k} (b-2) = 0$$

$$c+1 = 0 \Rightarrow c = -1$$

$$4 - a = 0 \Rightarrow a = 4$$

$$b - a = 0 \Rightarrow b = a$$
8 Find 'a' so that the vector
$$\vec{A} = (ax^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j} \text{ is irrotational.}$$
Soln:
Given: \vec{A} is irrotational.
$$\nabla \times \vec{A} = 0$$



(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & amp; Accredited by NBA (B.E - CSE, EEE, ECE, Mech & Comp; B.Tech.IT) COIMBATORE-641 035. TAMIL NADU

$$\frac{\partial \varphi}{\partial x} = y^{2} \cos x + z^{3} \Rightarrow \varphi = y^{2} \sin x + z^{3} + f(y,z)$$

$$\frac{\partial \varphi}{\partial y} = 2y \sin x - 4 \Rightarrow \varphi = y^{2} \sin x - 4y + f(x,z)$$

$$\frac{\partial \varphi}{\partial z} = 3xz^{2} \Rightarrow \varphi = y^{2}xz^{3} + f(x,y)$$

(3xz -y) \vec{k} is irrotational. Find φ such that $\vec{F} = \nabla \varphi$.

Soln: $\phi = 3x^2y + xz^3 - yz + c$

- (1) If $\nabla \varphi = yz\vec{i} + \chi z\vec{j} + \chi y\vec{k}$ then find φ .

 Soln: $\varphi = \chi yz + c$
- (12) Prove that $\operatorname{div} \hat{r} = 2/r$.

 Soln:

Soln:

$$div \hat{x} = \nabla \cdot \left(\frac{r}{r}\right)$$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right) \cdot \left(\frac{x\vec{i} + y\vec{j} + z\vec{k}}{r}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{r}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r}\right)$$

$$= \frac{1}{r} - \frac{1}{r^2} \cdot x \cdot \frac{\partial r}{\partial x} + \frac{1}{r} - \frac{1}{r^2} \cdot y \cdot \frac{\partial r}{\partial y} + \frac{1}{r^2} - \frac{1}{r^2} \cdot z \cdot \frac{\partial r}{\partial z}$$

$$= \frac{3}{r} - \frac{1}{r^2} \left[x \cdot \frac{\partial r}{\partial x} + y \cdot \frac{\partial r}{\partial y} + z \cdot \frac{\partial r}{\partial z}\right]$$

$$Now \quad r^2 = x^2 + y^2 + z^2$$

$$2r \cdot \frac{\partial r}{\partial x} = 2x \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}$$