



DEPARTMENT OF MATHEMATICS

GAUSS DIVERGENCE THEOREM :

If \vec{F} is a vector point function, finite and differentiable in a region R bounded by a closed surface S , then the surface integral of the normal component of \vec{F} taken over S is equal to the integral of divergence of \vec{F} taken over V .

$$\boxed{\iint_S \vec{F} \cdot \hat{n} \, ds = \iiint_V \nabla \cdot \vec{F} \, dv}$$

Where \hat{n} is the unit vector in the positive (outward drawn) normal to S .

Problems :

- ① Verify Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube $x=0, x=1, y=0, y=1, z=0, z=1$.

Soln: By Gauss divergence theorem,

$$\iint_S \vec{F} \cdot \hat{n} \, ds = \iiint_V \nabla \cdot \vec{F} \, dv$$

RHS :

$$\begin{aligned} \nabla \cdot \vec{F} &= \left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) \cdot (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \\ &= \frac{\partial}{\partial x} (4xz) + \frac{\partial}{\partial y} (-y^2) + \frac{\partial}{\partial z} (yz) \\ &= 4z - 2y + y \\ &= 4z - y \end{aligned}$$

$$\iiint_V \nabla \cdot \vec{F} \, dv = \int_0^1 \int_0^1 \int_0^1 (4z - y) \, dx \, dy \, dz = \frac{3}{2} \rightarrow \textcircled{1}$$

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LHS :

$$\iint_S \vec{F} \cdot \hat{n} \, ds$$

$$= \iint_{S_1} + \iint_{S_2} + \iint_{S_3} + \iint_{S_4} + \iint_{S_5} + \iint_{S_6}$$

Surface	\hat{n}	ds	Face equation
$S_1 - ABCD$	\vec{i}	$dy \, dz$	$x=1$
$S_2 - O EFG$	$-\vec{i}$	$dy \, dz$	$x=0$
$S_3 - B C E F$	\vec{j}	$dx \, dz$	$y=1$
$S_4 - O A D G$	$-\vec{j}$	$dx \, dz$	$y=0$
$S_5 - D C G H$	\vec{k}	$dx \, dy$	$z=1$
$S_6 - O A B E$	$-\vec{k}$	$dx \, dy$	$z=0$

$$\iint_{S_1} \vec{F} \cdot \hat{n} \, ds = \iint_{ABCD} (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \cdot \vec{i} \, dy \, dz$$

$$= \int_0^1 \int_0^1 4xz \, dy \, dz$$

$$= \int_0^1 \int_0^1 4z \, dy \, dz \quad (\because x=1)$$

$$= 2$$

$$\iint_{S_2} \vec{F} \cdot \hat{n} \, ds = \iint_{O EFG} (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \cdot (-\vec{i}) \, dy \, dz$$



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$$= \int_0^1 \int_0^1 (-4xz) dy dz = 0 \quad (\because x=0)$$

$$\iint_{S_3} \vec{F} \cdot \hat{n} ds = \iint_{BCEF} (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \cdot \vec{j} dx dz$$

$$= \int_0^1 \int_0^1 (-y^2) dx dz \quad (\text{Here } y=1)$$

$$= \int_0^1 \int_0^1 -dx dz = -1$$

$$\iint_{S_4} \vec{F} \cdot \hat{n} ds = \iint_{OADG} (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \cdot (-\vec{j}) dx dz$$

$$= \int_0^1 \int_0^1 y^2 dx dz = 0 \quad (\because y=0)$$

$$\iint_{S_5} \vec{F} \cdot \hat{n} ds = \iint_{DCGF} (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \cdot \vec{k} dx dy$$

$$= \int_0^1 \int_0^1 yz dx dy = \int_0^1 \int_0^1 y dx dy \quad (\because z=1)$$

$$= \int_0^1 y [x]_0^1 dy = \int_0^1 y dy = \left[\frac{y^2}{2} \right]_0^1 = \frac{1}{2}$$

$$\iint_{S_6} \vec{F} \cdot \hat{n} ds = \iint_{OABE} (-yz) dx dy = 0 \quad (\because z=0)$$

$$\iint_S \vec{F} \cdot \hat{n} ds = \iint_{S_1} + \iint_{S_2} + \iint_{S_3} + \iint_{S_4} + \iint_{S_5} + \iint_{S_6}$$

$$= 2 + 0 + (-1) + 0 + \frac{1}{2} + 0$$

$$= \frac{3}{2} \rightarrow \textcircled{2}$$

From ① & ②,

$$\iint_S \vec{F} \cdot \hat{n} ds = \iiint_V \nabla \cdot \vec{F} dv$$



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② Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$.

(or)

Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ and S is the surface of the rectangular parallelepiped bounded by $x=0$, $x=a$, $y=0$, $y=b$, $z=0$, $z=c$.

Soln:

By Gauss - divergence theorem,

$$\iint_S \vec{F} \cdot \hat{n} \, ds = \iiint_V \nabla \cdot \vec{F} \, dv.$$

RHS: $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$

$$\nabla \cdot \vec{F} = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot \left((x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k} \right)$$

$$= \frac{\partial}{\partial x} (x^2 - yz) + \frac{\partial}{\partial y} (y^2 - zx) + \frac{\partial}{\partial z} (z^2 - xy)$$

$$= (2x + 2y + 2z)$$

$$= 2(x + y + z)$$

$$\iiint_V \nabla \cdot \vec{F} \, dv = \int_0^a \int_0^b \int_0^c 2(x + y + z) \, dx \, dy \, dz$$

$$= 2 \int_0^a \int_0^b \left[xz + yz + \frac{z^2}{2} \right]_0^c \, dx \, dy$$

$$= 2 \int_0^a \int_0^b \left(xc + yc + \frac{c^2}{2} \right) \, dx \, dy$$