# Unit – III

## Two Dimensional Random Variable

* Joint Distribution
* Marginal Distribution
* Conditional Distribution
* Covariance Distribution
* Correlation Distribution
* Regression Distribution
* Functions of Random variable.

## Discrete

1. Joint Probability mass function

i). $P(x_i, y_j) \geq 0$

ii). $\sum_{i=1}^{n} \sum_{j=1}^{n} P(x_i, y_j) = 1$

2. To find constant:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} P(x_i, y_j) = 1$$

3. Marginal distribution function of $x$:

$$P(x) = \sum_{j=1}^{n} P(x_i, y_j)$$

Marginal distribution function of $y$:

$$P(y) = \sum_{i=1}^{n} P(x_i, y_j)$$

4. Cumulative distribution:

$$F(x, y) = P(x \leq x, y \leq y)$$

## Continuous

1. Joint Probability Density function

i). $f(x, y) \geq 0$

ii). $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dy\, dx = 1$

2. To find constant

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dy\, dx = 1$$

3. Marginal distribution function of $x$:

$$f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$$

Marginal distribution function of $y$:

$$f(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$$

4. Cumulative distribution:

$$F(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(x, y)\, dy\, dx$$

Discrete

5]. To check X & Y are Independent :

$$P(i,j) = P(X=i) \cdot P(Y=j)$$

6]. Conditional Distribution

$$P(X=x_i / Y=y_j) = \frac{P(X=x_i, Y=y_j)}{P(Y=y_j)}$$

$$P(Y=y_j / X=x_i) = \frac{P(X=x_i, Y=y_j)}{P(X=x_i)}$$

Continuous

5]. To check X & Y are Independent

$$f(x,y) = f(x) \cdot f(y)$$

6]. Conditional Distribution

$$f(x/y) = \frac{f(x,y)}{f(y)}$$

$$f(y/x) = \frac{f(x,y)}{f(x)}$$

7]. Joint cumulative Function is given then to find PDF

$$f(x,y) = \frac{\partial^2}{\partial x \partial y} F(x,y)$$

11. From the following table. for bivariate distribution of (x, y). Find

i). $P(x \leq 1)$ ii). $P(y \leq 3)$ iii). $P(x \leq 1, y \leq 3)$

iv). $P(x \leq 1 / y \leq 3)$ v). $P(y \leq 3 / x \leq 1)$

vi). Marginal distribution function of x & y.

vii). Conditional distribution of x given $y = 2$

viii). Estimate x & y are independent

ix). $P(x + y \leq 4)$

| Soln. x \ y → | 1 | 2 | 3 | 4 | 5 | 6 | P(x) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1/32 | 2/32 | 2/32 | 3/32 | → 8/32 |
| 1 | 1/16 | 1/16 | 1/8 | 1/8 | 1/8 | 1/8 | 10/16 |
| 2 | 1/32 | 1/32 | 1/64 | 1/64 | 0 | 2/64 | 8/64 |
| P(y) | ↓ 3/32 | 3/32 | 11/64 | 13/64 | 6/32 | 16/64 | 1 |

i). $P(X \leq 1)$

$$P(X \leq 1) = P(X=0) + P(X=1)$$
$$= \frac{8}{32} + \frac{10}{16}$$
$$= \frac{28}{32} = \frac{7}{8}$$

ii) $P(Y \leq 3)$

$$P(Y \leq 3) = P(Y=1) + P(Y=2) + P(Y=3)$$
$$= \frac{3}{32} + \frac{3}{32} + \frac{11}{64}$$
$$= \frac{23}{64}$$

$X = 0, 1$
$Y = 1, 2, 3$

iii). $P(X \le 1, Y \le 3)$

$P(X \le 1, Y \le 3) = P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)$

$= 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8}$

$= \frac{1+2+2+4}{32}$

$= \frac{9}{32}$

iv). $P(X \le 1 / Y \le 3) = \frac{P(X \le 1, Y \le 3)}{P(Y \le 3)}$

$= \frac{P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)}{P(Y=1) + P(Y=2) + P(Y=3)}$

$= \frac{0 + 0 + 1/32 + 1/16 + 1/16 + 1/8}{3/32 + 3/32 + 11/64}$

$= \frac{9/32}{23/64}$

$= \frac{18}{23}$

v). $P(Y \le 3 / X \le 1) = \frac{P(X \le 1, Y \le 3)}{P(X \le 1)}$

$= \frac{P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)}{P(X=0) + P(X=1)}$

$= \frac{0 + 0 + 1/32 + 1/16 + 1/16 + 1/8}{28/32}$

$$= \frac{9/32}{28/32}$$

$$= \frac{9}{28}$$

vi). marginal distribution function of x

| x | 0 | 1 | 2 |
|---|---|---|---|
| P(x) | 8/32 | 10/16 | 8/64 |

marginal distribution function of y:

| y | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| P(y) | 3/32 | 3/32 | 11/64 | 13/64 | 6/32 | 16/64 |

vii). Conditional distribution function of x on y = 2.

$P(x/y=2)$ :

$$P(x=0/y=2) = \frac{P(x=0, y=2)}{P(y=2)} = 0$$

$$P(x=1/y=2) = \frac{P(x=1, y=2)}{P(y=2)} = \frac{1/16}{3/32} = \frac{2}{3}$$

$$P/x=2/y=2) = \frac{P(x=2, y=2)}{P(y=2)} = \frac{1/32}{3/32} = 1/3$$

viii). x & y are independent.

$\Rightarrow P(x=i, y=j) = P(x=i) \cdot P(y=j)$

Consider $P(2,3)$

$$P(2,3) = P(x=2) \cdot P(y=3)$$

$$\frac{1}{64} \neq \frac{8}{64} \cdot \frac{11}{64}$$

$\therefore$ x & y are not independent.

iv). $P(X+Y \leq 4)$

$P(X+Y \leq 4) = P(0,1) + P(0,2) + P(0,3) + P(0,4) + P(1,1) + P(1,2) + P(1,3) + P(2,1) + P(2,2)$

$= 0 + 0 + 1/32 + 2/32 + 1/16 + 1/16 + 1/8 + 1/32 + 1/32$

$= \frac{1+2+2+2+4+1+1}{32}$

$= \frac{13}{32}$

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2). If the joint PDF of $(X, Y)$ is given by $P(x, y) = K(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find all the marginal probability distribution. Also find the prob. distribution of $(X+Y)$ and $P(X+Y > 3)$.

Soln.

Given $P(x, y) = K(2x + 3y)$

| Y \ X | 0 | 1 | 2 | P(Y) |
|---|---|---|---|---|
| 1 | 3K | 5K | 7K | 15K |
| 2 | 6K | 8K | 10K | 24K |
| 3 | 9K | 11K | 13K | 33K |
| P(X) | 18K | 24K | 30K | 72K |

Total probability = 1

$72K = 1$

$K = \frac{1}{72}$

| Y \ X → | 0 | 1 | 2 | P(Y) |
|---|---|---|---|---|
| 1 | 3/72 | 5/72 | 7/72 | ↓ 15/72 |
| 2 | 6/72 | 8/72 | 10/72 | 24/72 |
| 3 | 9/72 | 11/72 | 13/72 | 33/72 |
| P(X) → | 18/72 | 24/72 | 30/72 | 1 |

Marginal Probability function of X

$P(X=0) = 18/72$

$P(X=1) = 24/72$

$P(X=2) = \frac{30}{72}$

Marginal Probability function of Y

$P(Y=1) = \frac{15}{72}$

$P(Y=2) = \frac{24}{72}$

$P(Y=3) = \frac{33}{72}$

Probability Distribution of $X+Y$:

| | Probability |
|---|---|
| $P(X+Y=1)$ | $P(0,1) = 3/72$ |
| $P(X+Y=2)$ | $P(1,1) + P(0,2) = \frac{5}{72} + \frac{6}{72} = \frac{11}{72}$ |
| $P(X+Y=3)$ | $P(2,1) + P(1,2) + P(0,3) = \frac{7}{72} + \frac{8}{72} + \frac{9}{72} = \frac{24}{72}$ |
| $P(X+Y=4)$ | $P(1,3) + P(2,2) = \frac{11}{72} + \frac{10}{72} = \frac{21}{72}$ |
| $P(X+Y=5)$ | $P(2,3) = \frac{13}{72}$ |

$$P(x+y>3) = P(x+y=4) + P(x+y=5)$$

$$= \frac{21}{72} + \frac{13}{72}$$

$$= \frac{34}{72}$$

3). The two dimensional random variable $(x, y)$ has joint probability mass function $F(x, y) = \frac{x+2y}{27}$, $x = 0, 1, 2$; $y = 0, 1, 2$. Find the conditional distribution of $y$ for $x = x$. Also find conditional distribution of $y$ given $x = 1$.

Soln.

Given $F(x, y) = \frac{x+2y}{27}$

| $x \backslash y$ | 0 | 1 | 2 | $P(x)$ |
|---|---|---|---|---|
| 0 | 0 | 2/27 | 4/27 | 6/27 |
| 1 | 1/27 | 3/27 | 5/27 | 9/27 |
| 2 | 2/27 | 4/27 | 6/27 | 12/27 |
| $P(y)$ | 3/27 | 9/27 | 15/27 | 1 |

i). $P(y/x=x)$ ? ₱

When $x=0$,

$$P(y=0/x=0) = \frac{P(x=0, y=0)}{P(x=0)} = \frac{0}{6/27} = 0$$

$$P(y=1/x=0) = \frac{P(x=0, y=1)}{P(x=0)} = \frac{2/27}{6/27} = \frac{2}{6}$$

$$P(y=2/x=0) = \frac{P(x=0, y=2)}{P(x=0)} = \frac{4/27}{6/27} = \frac{4}{6}$$

when $x=1$,

$$P(y=0/x=1) = \frac{P(x=1, y=0)}{P(x=1)} = \frac{1/27}{9/27} = \frac{1}{9}$$

$$P(y=1/x=1) = \frac{P(x=1, y=1)}{P(x=1)} = \frac{3/27}{9/27} = \frac{3}{9}$$

$$P(y=2/x=1) = \frac{P(x=1, y=2)}{P(x=1)} = \frac{5/27}{9/27} = \frac{5}{9}$$

when $x=2$,

$$P(y=0/x=2) = \frac{P(x=2, y=0)}{P(x=2)} = \frac{2/27}{12/27} = \frac{2}{12}$$

$$P(y=1/x=2) = \frac{P(x=2, y=1)}{P(x=2)} = \frac{4/27}{12/27} = \frac{4}{12}$$

$$P(y=2/x=2) = \frac{P(x=2, y=2)}{P(x=2)} = \frac{6/27}{12/27} = \frac{6}{12}$$

ii). $P(y/x=1)$

$$P(y=0/x=1) = \frac{1}{9}$$

$$P(y=1/x=1) = \frac{3}{9}$$

$$P(y=2/x=1) = \frac{5}{9}$$