Common Relationships

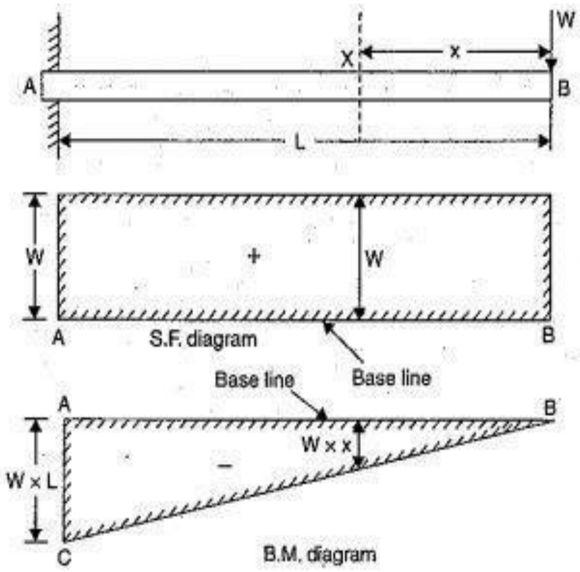
Load	0	Constant	Linear
Shear	Constant	Linear	Parabolic
Moment	Linear	Parabolic	Cubic

Cantilever beam with a point load at its free end

Consider a cantilever beam AB of length *l*, fixed at A and free at B, and carrying a point load W at the free end B.

Calculation for SFD:

Take a section *X-X* at a distance of *x* from free end B.



Consider a right portion of the section.

- The shear force at this section is equal to resultant force acting on the right portion at given section.
- But the resultant force acting on the right portion at the section *X-X* is *W* and acting in downward direction.
- Force on right portion acting downward is considered positive.
- Hence shear force at section *X-X* is positive. SF at section *X-X* = +*W*
- There is no other load between A & B. So that Shear Force will be constant at all sections of cantilever beam.

Calculation for BMD:

Bending moment at section $X\text{-}X = M_x = -W.x$ ……………………………… *(i)*

- Bending moment will be negative as for the right portion of the section, the moment of W at X-X is clockwise.
- Bending of cantilever will take place in such a manner that convexity will be at the top of the beam.
- From equation (i) it is clear that BM of a cantilever beam at any section is proportional to the distance of the section from the free end

  BM at point $A_{(x=0)} = 0$

  BM at Point $B_{(x=l)} = -W.l$
- Hence bending moment follows straight line for such cases.

Q. A cantilever beam of length 2m carries the point loads as shown in figure. Draw shear force and bending moment diagram for cantilever beam.

Calculation for SFD:

SF at point D = 800N

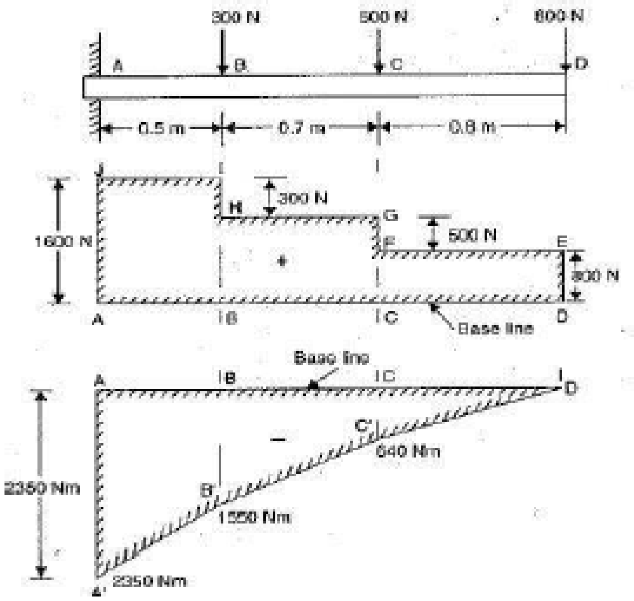
SF just right to C = 800N

Sf at point C = 800+500 = 1300N

SF just right to B = 1300N

SF at point B = 1300+300 = 1600N

Sf just right to A = 1600N



Calculation for BMD:

BM at point D = 0

BM at point C = −800×0.8 = −640N-m

BM at point B = −800×1.5−500×0.7

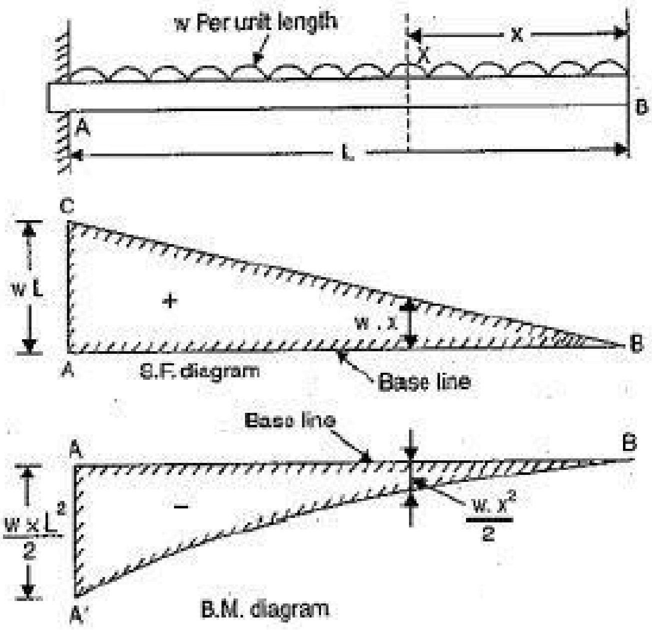
= −1550N-m

BM at point A = −800×2−500×1.2−300×0.5

= −2350N-m

Cantilever with Uniformly distributed load

Consider a beam AB of *l* length fixed at A and carrying a uniformly distributed load of *w* per unit length over the entire length of cantilever.



The SF at section *X-X* will be equal to the resultant force acting on the right portion of the section. Resultant force on right portion = *w.x*

The resultant force is acting downwards.

Q. A cantilever of length 2m carries a uniformly distributed load of 1kN/m run over a length of 1.5m from the free end. Draw the shear force and bending moment diagram for the cantilever.

Calculation for SFD:

SF at section *X-X* = *w.x*

SF at point $B_{(x=0)}$ = 0

SF at point $A_{(x=l)}$ = *w.l*

Calculation for BMD:

As we discussed earlier that the UDL over a section is converted into point load acting at the C.G. of the section

BM at section *X-X* = $-w.\,x.\,\frac{x}{2} = -w\,\frac{x^2}{2}$

BM at Point $B_{(x=0)}$ = *0*

BM at point $A_{(x=l)}$ = $-\,w\,\frac{l^2}{2}$

Calculation for SFD:

SF at section *X-X* between B & C at a distance of x from free end = $w.x = x$

SF at point $B_{(x=0)} = 0$

SF at point $C_{(x=1.5)} = 1.5kN$

SF just right to A = 1.5kN

Calculation for BMD:

BM at section *X-X* between B & C at a distance of x from free end = $-\ w.x.\frac{x}{2} = -\ \frac{x^2}{2}$

BM at point $B_{(x=0)} = 0$

*B*M at point $C_{(x=1.5)} = -\frac{1.5^2}{2} = -1.125kN\text{-}m$

BM at section *Y-Y* between A & C at a distance of *y* from free end

$= -1 \times 1.5 \times [\frac{1.5}{2} + (x - 1.5)] = -1.5\ (x - 0.75)$

[Total load due to UDL is = 1×1.5 = 1.5kN

This load will act at a distance = $\frac{1.5}{2} = 0.75m$]

BM at point $C_{(x=1.5)} = -1.5\ (1.5 - 0.75)$

$= -1.125kN\text{-}m$

BM at point $A_{(x=2)} = -1.5\ (2 - 0.75)$

$= -1.875kN\text{-}m$

Q. A cantilever of length 2m carries a UDL of 1.5kN/m run over the whole length and a point load of 2kN at a distance of 0.5m from the free end. Draw SF and BM diagram.

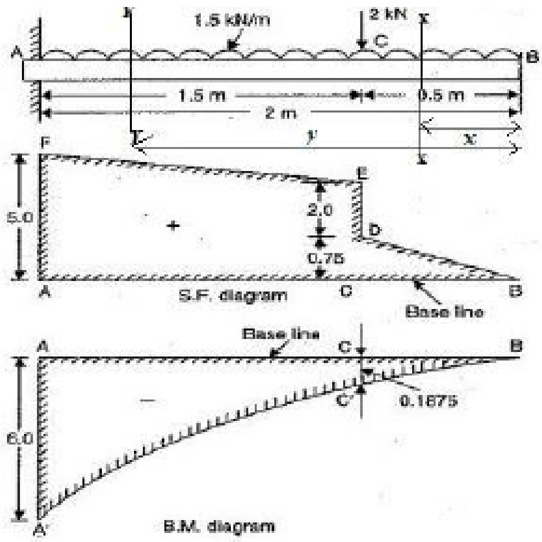
Calculation for SFD:

SF at section *X-X* between B & C at a distance of x from free end = $w.x = 1.5x$

SF at point $B_{(x=0)} = 0$
SF just right to $C_{(x=0.5)} = 1.5 \times 0.5 = 0.75$kN

BM at any section *Y-Y* between A & C at a distance y from B (free end) is given by

BM at section *Y-Y* = $(1.5 \times y + 2)$ kN
SF at point $C_{(y=0.5)} = 0.75 + 2 = 2.75$kN
SF just right to $A_{(y=2)} = 5$kN



Calculation for BMD:

BM at section *X-X* between B & C at a distance of x from free end = $-w.x \cdot \frac{x}{2}$

$$= -1.5\frac{x^2}{2} = -0.75x^2$$

BM at point $B_{(x=0)} = 0$
BM at point $C_{(x=0.5)} = -0.75 \times 0.5^2$
$= -0.1875$kN-m
BM at section *Y-Y* between A & C at a distance of *y* from free end
$$= -1.5y.\frac{y}{2} - 2(y-0.5)$$
$$= -0.75y^2 - 2$$
$(y-0.5)$ *BM* at point $C_{(y=0.5)}$
$= -0.75 \times 0.5^2$
$-2(0.5-0.5)$
$=$
-0.1875kN-m

BM at section *Y-Y* between A & C at a distance of *y* from free end
$$= -1.5y.\frac{y}{2} - 2(y-0.5)$$
$$= -0.75y^2 - 2$$
$(y-0.5)$ *BM* at point $C_{(y=0.5)}$
$= -0.75 \times 0.5^2$
$-2(0.5-0.5)$
$=$
-0.1875kN-m
BM at point $A_{(y=2)} = -0.75 \times 2^2 - 2(2-0.5)$
$= -6$kN-m

Q. A cantilever 1.5m long is loaded with a uniformly distributed load of 2kN/m run over a length of 1.25m from the free end. It also carries a point load of 3kN at a distance of 0.25m from the free end. Draw SFD & BMD of cantilever.