SF at point $C_{(x=\frac{l}{2})}$ = $\frac{wl}{4} - \frac{wl^2}{4l} = 0$

SF just left to $B_{)}$ = $-\frac{wl}{4}$

Calculation for BMD:

BM at section X-X = $R_A \times x$ – load on length AF× $Centre\ of\ Gravity\ of\ triangle\ AFD$

$$= \frac{Wl}{4} \times x - \frac{wx^2}{l} \times \frac{x}{3}$$

$$= \frac{Wlx}{4} - \frac{wx^3}{3l}$$ {It showing cubic equation}

BM at point $A_{(x=0)} = 0$

BM at point $C_{(x=\frac{l}{2})}$ = $\frac{wl}{4} \times \frac{l}{2} - \frac{w}{3l} \times (\frac{l}{2})^3 = 0$

$$= \frac{wl^2}{8} - \frac{wl^2}{24} = \frac{wl^2}{12}$$

$$= \frac{Wl}{6}$$ {where W = $\frac{Wl}{2}$ }

BM just left to B = 0

***NOTE** – Bending moment is always maximum, where Shear Force become zero after changing its sign*

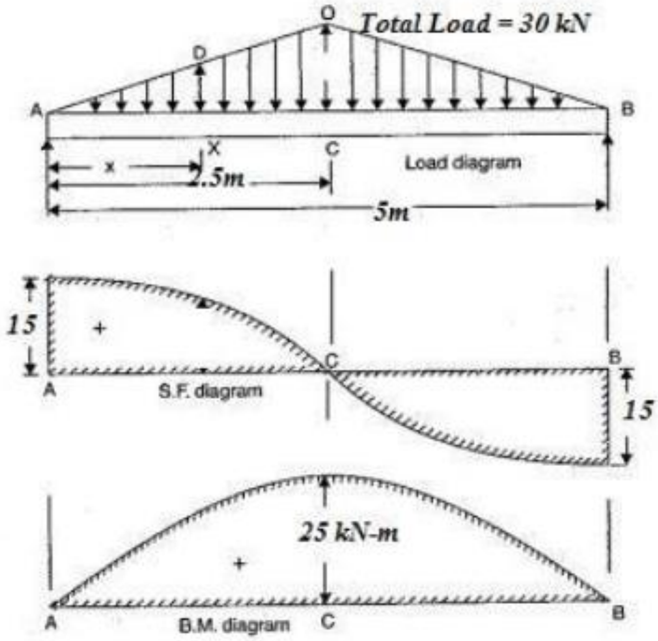
***NOTE** – In case of simply supported beam, bending moment will always be zero at both ends (support)*

Q. A simply supported beam of 5m span carries a triangular load of 30kN, Draw SFD and BMD for the beam.

Calculation for reaction:

$R_A = R_B$ = Half of the total load

$R_A = R_B = 15$ kN



Calculation for SFD:

SF at point A = 15 kN

SF at point C = 15 $-(30/2) = 0$

SF just left to B = – 15 kN

Calculation for BMD:

BM at point A = $R_A \times 0 = 0$

BM at point C = $15\times2.5 - 15 \times (2.5/3) =$ 25 kN-m

BM at point B = $15\times5 - (30/2) \times 5 = 0$kN-m

Simply Supported beam with a gradually varying load from zero at one end to w per unit length at other end

Consider a simply supported beam AB of length l and carrying a gradually varying load from 0 at A to w per unit length at B

Calculation for reaction:

Total load on beam = Area of triangle ABE

$= \frac{1}{2} \times base \times$ height

$= \frac{1}{2} \times w \times l$

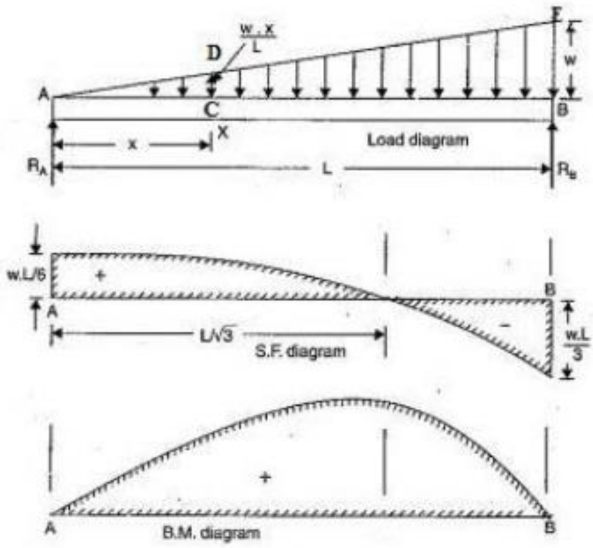
$R_A + R_B = \frac{wl}{2}$

$\Sigma M_A = 0$

$R_A \times 0 + l \times R_B = \frac{wl}{2} \times \frac{2l}{3}$

$R_B = \frac{wl}{3}$

$R_A = \frac{wl}{6}$



Consider any section X-X between A & B at a distance x from A

Rate of loading at section X-X = DC

ΔABE and ΔACD are two similar triangles

$$\frac{EB}{AB} = \frac{DC}{AC} \qquad \frac{w}{l} = \frac{DC}{x} \qquad DC = \frac{wx}{l}$$

Calculation for SFD:

SF at section X-X = R_A – load on length AC

$$= \frac{Wl}{6} - \frac{1}{2} \times x \times \frac{wx}{l}$$ {It showing parabolic equation}

$$= \frac{Wl}{6} - \frac{wx^2}{2l}$$

SF at point $A_{(x=0)} = \frac{Wl}{6}$

SF just left to B $= \frac{Wl}{6} - \frac{wl^2}{2l} = -\frac{Wl}{3}$

We know that Maximum bending moment will occur at that point where SF changes its sign (SF = 0)

SF at section X-X between A & B at a distance of *x* from A

$\frac{Wl}{6} - \frac{wx^2}{2l} = 0$ $\quad x = \frac{l}{\sqrt{3}}$

Or x = 0.577 l

Calculation for BMD:

BM at section X-X = $R_A \times x$ – load on length AC$\times$ $Centre\ of\ Gravity\ of\ triangle\ ACD$

$$= \frac{Wl}{6} \times x - \frac{1}{2} \times x \times \frac{wx}{l} \times \frac{x}{3}$$

$$= \frac{Wlx}{6} - \frac{wx^3}{6l}$$ {It showing cubic equation}

BM at point $A_{(x=0)} = 0$

BM at point C $= \frac{Wl.l}{6} - \frac{wl^3}{6l} = 0$

Calculation for Maximum Bending Moment:

BM will be maximum at that point where Shear Force is zero, i.e. at $x = \frac{1}{\sqrt{3}}$

Maximum bending moment $(\text{at} = \frac{l}{\sqrt{3}}) = \frac{Wl}{6} \times \frac{l}{\sqrt{3}} - \frac{w}{6l} \times \left[\frac{l}{\sqrt{3}}\right]^3 = \frac{wl^2}{9\sqrt{3}}$

Q.A simply supported beam of length 5m carries a uniformly increasing load of 800N/m run at one end to 1600N/m run at other end. Draw the shear force and bending moment diagram for the beam, also calculate the position & magnitude of maximum bending moment.