

We know that when a beam is under bending the fibres at the top will be lengthened while at the bottom will be shortened provided the bending moment M acts at the ends. In between these there are some fibres which remain unchanged in length that is they are not strained, that is they do not carry any stress. The plane containing such fibres is called neutral surface.

The line of intersection between the neutral surface and the transverse exploratory section is called the neutral axis **(N A)** .

Bending Stresses in Beams or Derivation of Elastic Flexural formula :

In order to compute the value of bending stresses developed in a loaded beam, let us consider the two cross-sections of a beam **HE** and **GF** , originally parallel as shown in fig 1(a).when the beam is to bend it is assumed that these sections remain parallel i.e.**H'E'** and **G'F'** , the final position of the sections, are still straight lines, they then subtend some angle ϕ .

Consider now fiber **AB** in the material, at a distance y from the N.A, when the beam bends this will stretch to **A'B'**

Therefore ,

$$\text{strain in fibre AB} = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{A'B' - AB}{AB}$$

But $AB = CD$ and $CD = C'D'$

refer to fig1(a) and fig1(b)

$$\therefore \text{strain} = \frac{A'B' - C'D'}{C'D'}$$

Since CD and $C'D'$ are on the neutral axis and it is assumed that the Stress on the neutral axis zero. Therefore, there won't be any strain on the neutral axis

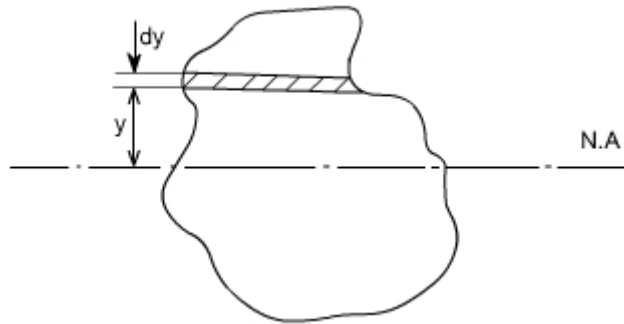


$$= \frac{(R + y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = \frac{y}{R}$$

However $\frac{\text{stress}}{\text{strain}} = E$ where $E = \text{Young's Modulus of elasticity}$

Therefore, equating the two strains as obtained from the two relations i.e.,

$$\frac{\sigma}{E} = \frac{y}{R} \text{ or } \frac{\sigma}{y} = \frac{E}{R} \quad \dots\dots\dots(1)$$



Consider any arbitrary a cross-section of beam, as shown above now the strain on a fibre at a distance 'y' from the N.A, is given by the expression

$$\sigma = \frac{E}{R} y$$

if the shaded strip is of area 'dA'
then the force on the strip is

$$F = \sigma \delta A = \frac{E}{R} y \delta A$$

$$\text{Moment about the neutral axis would be } = F \cdot y = \frac{E}{R} y^2 \delta A$$

The total moment for the whole cross-section is therefore equal to

$$M = \sum \frac{E}{R} y^2 \delta A = \frac{E}{R} \sum y^2 \delta A$$

Now the term $\sum y^2 \delta A$ is the property of the material and is called as a second moment of area of the cross-section and is denoted by a symbol I.

Therefore

$$M = \frac{E}{R} I \quad \dots\dots\dots(2)$$

combining equation 1 and 2 we get

$$\boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}}$$

This equation is known as the Bending Theory Equation.The above proof has involved the assumption of pure bending without any shear force being present.