Boats and Streams

Boats and streams is an application of concepts of speed, time and distance. Speed of river flowing either aides a swimmer (boat), while travelling with the direction of river or it opposes when travelling against the direction of river.

Still water: If the speed of water of a river is zero, then water is considered to be still water.

Stream water: If the water of a river is moving at a certain speed, then it is called as stream water.

Speed of Boat: Speed of boat means speed of boat (swimmer) in still water. In other words, if the speed of a boat (swimmer) is given, then that particular speed is the speed in still water.

Downstream Motion: If the motion of a boat (swimmer) is along the direction of stream, then such motion is called downstream motion.

Upstream Motion: If the motion of a boat (swimmer) is against the direction of stream, then such motion is called upstream motion.

Basic Formulae Related to Boats and Streams

If the speed of a boat in still water is $x\ km/h$ and speed of the stream is $y\ km/h$, then

1. Speed downstream = $(x+y)km/h$
2. Speed upstream = $(x-y)\ km/h$
3. Speed of a boat in still water $(x) = \frac{1}{2}$ (Speed downstream + Speed upstream)
4. Speed of a stream $(y) = \frac{1}{2}$ (Speed downstream – Speed upstream)

Example : 1 A man can row with a speed of 6 km/h in still water. What will be his speed with the stream, if the speed of stream is 2 km/h?

Solution: Given, speed of man in still water = $x = 6\ km/h$ and speed of stream = y=2 km/h

Speed downstream = $x + y = 6 + 2 = 8\ km/h.$

Example : 2 If the speed of a boat in still water is 8 km/h and the rate of stream is 4 km/h, then find upstream speed of the boat.

Solution: Given, speed of a boat = $x = 8\ km/h$

Speed of stream = $y = 4\ km/h$

Speed upstream = $x - y = 8 - 4 = 4\ km/h$

Example : 3 Shantanu can row upstream at 10 km/h and downstream at 18 km/h. Find the man's rate in still water and the rate of the current.

Solution: Speed upstream = 10 km/h and Speed downstream = 18 km/h

According to the formula,

Man's rate in still water = $\frac{1}{2}(Speed\ downstream + Speed\ upstream)$

$$=\frac{1}{2}(18+10) = \frac{28}{2} = 14\ km/h$$

Speed of current = $\frac{1}{2}(Speed\ downstream - Speed\ upstream)$

$$=\frac{1}{2}(18-10) = \frac{8}{2} = 4\ km/h$$

Example : 4 What time will be taken by a boat to cover a distance of 64 km along the stream, if speed of boat in still water is 12 km/h and speed of stream is 4 km/h?

Solution: Given that, distance = 64 km, speed of boat in still water $= x = 12\ km/h$ and speed of stream $= y = 4\ km/h$

Downstream speed of boat $= x + y = 12 + 4 = 16\ km/h$

Required time = $\frac{\text{Distance}}{\text{Speed(downstream)}} = \frac{64}{16} = 4\ h.$

Example : 5 A boat takes 8 h to row 48 km downstream and 12 h to row the same distance upstream. Find the boat's rate in still water and rate of current.

Solution: Speed downstream = $\frac{Distance}{Time} = \frac{48}{8} = 6\ km/h$

Speed upstream = $= \frac{48}{12} = 4\ km/h$

Now, rate of boat in still water = $\frac{(\text{Speed downstream}+\text{Speed upstream})}{2} = \frac{6+4}{2} = 5\ km/h$

and rate of current = $\frac{(\text{Speed downstream}-\text{Speed upstream})}{2} = \frac{6-4}{2} = 1\ km/h.$

Fast Track Techniques to solve the questions:

Technique :1

If speed of stream is a and a boat (swimmer) takes n times as long to row up as to row down the river, then

Speed of boat (Swimmer) in still water = $\frac{a(n+1)}{(n-1)}$

Note : This formula is applicable for equal distances.

Example : 6 Rajnish can row 12 km/h in still water. It takes him twice as long to row up as to row down the river. Find the rate of stream.

Solution: Here speed of Rajnish in still water = 12 km/h

$n = 2$, Speed of stream (a) =?

According to the formula, Speed in still water = $\frac{a(n+1)}{(n-1)}$

$$12 = \frac{a(2+1)}{(2-1)}$$

$$3a = 12 \Rightarrow a = 4\ km/h$$

Technique : 2

A person can row at a speed of x in still water. If stream is flowing at a speed of y, it takes time T to row to a place and back, then

Distance between two places = $\frac{T(x^2-y^2)}{2x}$

Example : 7 A man can row 12 km/h in still water. When the river is running at 2.4 km/h, it takes him 1 h to row to a place and to come back. How far is the place?

Solution: Here, Speed of man in still water = $x = 12\ km/h$

Speed of river = $= 24\ km/h$; $T = 1\ h$

According to the formula,

Required distance = $\frac{T(x^2-y^2)}{2x} = \frac{1\times[(12)^2-(2.4)^2]}{2\times 12} = \frac{138.24}{24} = 5.76\ km$

Technique : 3

A man rows a certain distance downstream in x h and returns the same distance in y h. when the stream flows at the rate of a km/h, then

Speed of the man in still water = $\frac{a(x+y)}{(y-x)}$

Example : 8 Kamal can row a certain distance downstream in 12 h and can return the same distance in 18 h. If the stream flows at the rate of 6 km/h, then find the speed of Kamal in still water.

Solution: Here, $x = 12\ h, y = 18\ h$

Rate of stream (a) = 6 km/h

According to the formula,

Speed of kamal in still water = $\frac{a(x+y)}{(y-x)} = \frac{6\times(12+18)}{(18-12)} = \frac{6\times 30}{6} = 30\ km/h$

Note: If in case of technique 3, man's speed in still water is b km/h and we are asked to find the speed of stream, then technique 3 takes the form as

Speed of the stream = $\frac{b(y-x)}{(x+y)}$

Example : 9 If in the above example, the speed of Kamal in still water is 12 km/h, then find the speed of the stream.

Solution: Here, b=12, y=18 and x=12

According to the formula,

Speed of stream = $\frac{b(y-x)}{(x+y)} = \frac{12(18-12)}{18+12} = \frac{12\times 6}{30} = \frac{12}{5} = 2.4\ km/h$

Technique : 4

If boat's (swimmer's) speed in still water is a km/h and river Is flowing with a speed of b km/h, then average speed in going to a certain place and coming back to starting point is given by $\frac{(a+b)(a-b)}{a}\ km/h.$

Example : 10 Ramesh rows in still water with a speed of 4.5 km/h to go to a certain place and to come back. Find his average speed for the whole journey, if the river is flowing with a speed of 1.5 km/h.

Solution: Here a= 4.5 km/h , b= 1.5 km/h

Average speed = $\frac{(a+b)(a-b)}{a} = \frac{(4.5+1.5)(4.5-1.5)}{4.5} = \frac{6\times 3}{4.5} = \frac{18}{4.5} = 4\ km/h$

Technique : 5

When boat's speed (swimmer's speed) in still water is a km/h and river is flowing with a speed of b km/h and time taken to cover a certain distance upstream is T more than the time taken to cover the same distance downstream, then Distance = $\frac{(a^2-b^2)T}{2b}$

Example : 11 A boat's speed in still water is 10km/h, while river is flowing with a speed of 2 km/h and time taken to cover a certain distance upstream is 4 h more than time taken to cover the same distance downstream. Find the distance.

Solution: Here, a = 10 km/h , b= 2 km/h and T=4 h

According to the formula,

Required distance = $\frac{(a^2-b^2)}{2b} \times T = \frac{(10^2-2^2)}{2\times 2} \times 4 = \frac{100-4}{4} \times 4 = 100 - 4 = 96\ km$

Technique : 6

If a man covers l km distance in $t_1\ h$ along the direction of river and he covers same distance in $t_2\ h$ against the direction of river, then

Speed of man = $\frac{1}{2}\left(\frac{l}{t_1} + \frac{l}{t_2}\right) = \frac{l}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$

Speed of the stream = $\frac{1}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)=\frac{l}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)$

Example : 12 A boat covers 20 km in an hour with downstream and covers the same distance in 2 h with upstream. Then, find the speed of boat in still water and speed of stream.

Solution: Here, l=20 km, $t_1 = 1\ h, t_2 = 2\ h$

Speed of boat in still water = $\frac{l}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}\right)=\frac{20}{2}\left(\frac{1}{1}+\frac{1}{2}\right)=10\times\frac{3}{2}=15\ km/$h

Speed of stream== $\frac{l}{2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)=\frac{20}{2}\left(\frac{1}{1}-\frac{1}{2}\right)=10\times\frac{1}{2}=5\ km/$h

Pipes and Cisterns

Problems on Pipes and Cisterns are based on the basic concept of time and work. Pipes are connected to a tank or cistern and are used to fill or empty the tank or cistern. In pipe and cistern, the work is done in form of filling or emptying a cistern/tank.

Inlet pipe : It fills a tank/cistern/reservoir.

Outlet pipe: It empties a tank/cistern/reservoir.

Important Points:

1. If a pipe can fill/empty a tank in 'm' h, then the part of tank filled /emptied in $1\ h=\frac{1}{m}$.

For example, A pipe can fill the tank in 7 h, then the volume of tank filled in $1\ h=\frac{1}{7}$.

2. If a pipe can fill/empty '$\frac{1}{m}$' part of a tank in 1 h, then it can fill/empty the whole tank in $'m'$ h.

For example, If a pipe can fill $\frac{1}{5}$ part of a tank in 1 h, then it can fill the whole tank in 5 h.

3. Generally, time taken to fill a tank is taken positive (+ ve) and time taken to empty a tank is taken negative (- ve).
4. If a pipe fills a tank in m h and an another pipe fills in n h. Then, part filled by both pipes in $1\ h=\frac{1}{m}+\frac{1}{n}$.

Example : 1 An outlet pipe can empty a cistern in 5 h. In what time will the pipe empty $\frac{2}{5}$ part of the cistern?

Solution : Time taken to empty full cistern = 5 h

Time taken to empty $\frac{2}{5}$ part of the cistern = $\frac{2}{5} \times 5 = 2\ h$

Example : 2 If a pipe can fill a tank in 2 h and another pipe can fill the same tank in 6 h, then what part of a tank will be filled by both the pipes in 1 h, if they are opened simultaneously?

Solution : In 1 h, part filled by 1st pipe = $\frac{1}{m} = \frac{1}{2}$

In 1 h, part filled by 2nd pipe = $\frac{1}{n} = \frac{1}{6}$.

In 1 h, part filled by both the pipes together = $\left(\frac{1}{m} + \frac{1}{n}\right) = \left(\frac{1}{2} + \frac{1}{6}\right) = \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3}$ part

Example : 3 If a pipe can fill a tank in 5 h and an another pipe can empty the tank in 10 h, then part fill by both pipes in 1 h, if both pipes are open simultaneously.

Solution: In 1 h, part filled by 1st pipe = $\frac{1}{m} = \frac{1}{5}$

In 1 h, part emptied by 2nd pipe = $\frac{1}{n} = \frac{1}{10}$

In 1 h, part filled by both pipes when open simultaneously = $\frac{1}{m} - \frac{1}{n}$

$= \frac{1}{5} - \frac{1}{10} = \frac{2-1}{10} = \frac{1}{10}$ part [-ve sign is used, as 2nd pipe empties the tank]

Fast Track Techniques to solve the Questions

Technique :1

If a pipe can fill/empty a tank in $'m'$ h and an another pipe can fill/empty the same tank in $'n'$ h, then

(i) If both pipes either fills or empties the tank, then the time taken to fill or empty the tank when both pipes are opened is $t = \frac{mn}{m+n}$

(ii) If first pipe fills the tank and second pipe empties the tank, then the time taken to fill the tank when both pipes are opened is $t = \frac{mn}{m-n} : m > n$

(iii) If first pipe fills the tank and second pipe empties the tank, then the time taken to empty the tank when both pipes are opened is $t = \frac{mn}{n-m} : n > m$

Example : 4 Two pipes A and B can fill a tank in 18 h and 12 h, respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Solution: Time taken by both pipes to fill the tank= $\frac{mn}{m+n}$ where m and n are the time taken to fill the tank by individual pipes.

Here,$m = 18, n = 12$

Time taken to fill the tank $= \frac{m \times n}{m+n} = \frac{18\times12}{18+12} = \frac{18\times12}{30} = \frac{3\times12}{5} = \frac{36}{5} = 7\frac{1}{5}\ h$

Example : 5 A pipe can fill a tank in 5 h, while another pipe can empty it in 6 h. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Solution : Here, m= 5 h and n = 6 h

Time taken to fill the tank = $\frac{m\times n}{n-m} = \frac{5\times6}{6-5} = \frac{30}{1} = 30\ h$

Example : 6 A pipe can fill a tank in 10 h. Due to a leak in the bottom, it fills the tank in 20 h. If the tank is full, how much time will the leak take to empty it?

Solution: Here, $m = 10$ and $n = 20$

According to the formula,

Required time taken to empty the tank = $\frac{m\times n}{n-m} = \frac{10\times20}{20-10} = \frac{200}{10} = 20\ h$

Technique :2

If three pipes can fill a tank separately in m,n and p h, respectively, then part of tank filled in 1 h by all the three pipes is given by $\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{p}\right)$ and total time taken to fill the tank is given by $\frac{mnp}{np+mp+mn}h$.

Note : If any one of the three pipes is used to empty the tank, then time taken by that particular pipe will be negative (-ve). Suppose, 3^{rd} pipe is used to empty the tank. Then, the above formulae takes the form as $\left(\frac{1}{m}+\frac{1}{n}-\frac{1}{p}\right)$ and $\frac{mnp}{np+mp-mn}h$.

Example : 7 Three pipes m, n and p can fill a tank separately in 4,5 and 10 h, respectively. Find the time taken by all the three pipes to fill the tank when the pipes are opened together.

Solution : Part filled by pipe m in 1 h = $\frac{1}{4}$

Part filled by pipe n in 1 h = $\frac{1}{5}$

Part filled by $(m + n + p)$ pipes in 1 h = $\frac{1}{4}+\frac{1}{5}+\frac{1}{10} = \frac{5+4+2}{20} = \frac{11}{20}$

Required time to fill the tank = $\frac{20}{11}h = 1\frac{9}{11}h$.

Example : 8 Pipe A can fill a tank in 20 h while pipe B alone can fill it in 10 h and pipe C can empty the full tank in 30 h. If all the pipes are opened together, how much time will be needed to male the tank full?

Solution : Here, $m = 20, n = 10$ and p=30

Required time to fill the tank = $\frac{mnp}{np+mp-mn} = \frac{20\times10\times30}{10\times30+20\times30-20\times10}$

$$= \frac{6000}{300+600-200} = \frac{6000}{700} = \frac{60}{7} = 8\frac{4}{7}h.$$

Technique : 3

Two pipes A and B together can fill a tank in time t. If time taken by A alone is more than t by a and time taken by B alone is more than t by b, then $t = \sqrt{ab}$.

Example : 9 Two pipes A and B are opened together to fill a tank. Both the pipes fill the tank in time t. If A separately takes 4 min more time than t to fill the tank and B takes 64 min more than t to fill the tank, find the value of t.

Solution : We know that, time taken by both pipes to fill the tank (t) = $\sqrt{ab}$

Here a=4 and b= 64

$t = \sqrt{4 \times 64} = 2 \times 8 = 16$ min

Technique : 4

A full tank get emptied in $'a'$ h due to presence of a leak in it. If a tap which fills it at a rate of $'b'\ L/h$, is opened, then it get emptied in $'c'$ h.

Therefore, volume of tank = $\frac{abc}{c-a}$

Example : 10 A full tank get emptied in 6 min due to presence of an orifice in it. On opening a tap which can fill the tank at the rate of 8 L/min, the tank get emptied in 10 min. Find the capacity of tank.

Solution: Here, $a = 6, b = 8$ and $c = 10$

Capacity of tank $= \frac{abc}{c-a} = \frac{6\times8\times10}{10-6} = 120\ L.$

Technique : 5

If two taps A and B, which can fill a tank, such that efficieny of A is n times of B and takes t min less/more than B to fill the tank, then

(i) Time taken to fill the tank by both pipes together = $\frac{nt}{n^2-1}\ min$

(ii) Time taken to fill the tank by faster tap = $\frac{t}{n-1}\ min$

(iii) Time taken to fill the tank by slower tap = $\frac{nt}{n-1}\ min$

Example : 11 If tap A can fill a tank 3 times faster than tap B and takes 28 min less than tap B to fill the tank. If both the taps are opened simultaneously, then find the time taken to fill the tank.

Solution: Here, n = 3 and t = 28

According to the formula,

So, time taken to fill the tank by both pipes together = $\frac{nt}{n^2-1} = \frac{28\times 3}{(3)^2-1} = \frac{21}{2}\ min$

Technique : 6

Two pipes A and B can fill a tank in x min and y min, respectively. If both the pipes are opened simultaneously, then the time after which pipe B should be closed so that the tank is full in t min, is $\left[y\left(1-\frac{t}{x}\right)\right]\ min.$

Example : 12 Two pipes A and B can fill a tank in 12 and 16 min, respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 9 min?

Solution: Here, $x = 12, y = 16$ and $t = 9$

Required time after which B should be closed = $y\left(1-\frac{t}{x}\right) = 16\left(1-\frac{9}{12}\right) = 16 \times \frac{3}{2} = 4\ min.$