

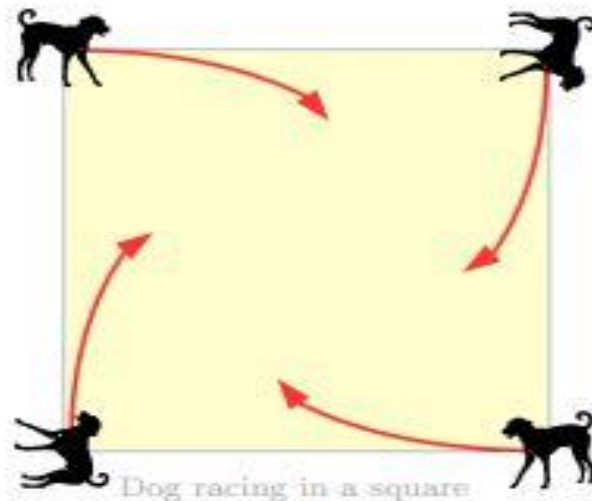


DEPARTMENT OF MATHEMATICS

UNIT II ORDINARY DIFFERENTIAL EQUATIONS

Four dogs are positioned at the corners of a square ($d=1\text{ m}$), chase each other in clockwise direction with the same constant speed .

As their target is moving, they will follow a curved path, eventually colliding in the center of the square.



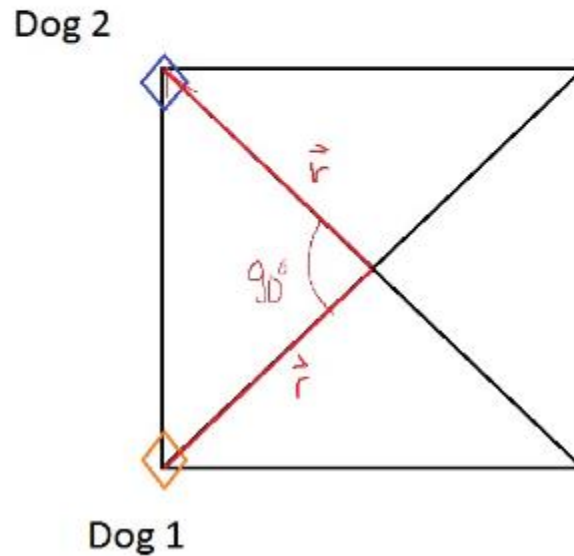
- Why is the total length of the path just 1 m ?
- Find and solve a differential equation for the radius $r(\theta)$ in polar coordinates.

Solution:

$$r(\theta) = a + b\theta \quad \text{or} \quad r(\theta) = a\theta^{\frac{1}{n}}$$

If *Dog 1* is positioned at $(r, \theta) \implies$ *Dog 2* is positioned at $(r, \theta + \frac{\pi}{2})$

Picture:



$$x_1 = r \cos(\theta)$$

$$y_1 = r \sin(\theta)$$

$$x_2 = r \cos\left(\theta + \frac{\pi}{2}\right) = -r \sin(\theta)$$

$$y_2 = r \sin\left(\theta + \frac{\pi}{2}\right) = r \cos(\theta)$$

If these are the two position vectors then the vector joining the two points is my velocity vector.

$$\begin{aligned} \implies \frac{dy}{dx} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{r \sin\left(\theta + \frac{\pi}{2}\right) - r \sin(\theta)}{r \cos\left(\theta + \frac{\pi}{2}\right) - r \cos(\theta)} = \frac{\sin\left(\theta + \frac{\pi}{2}\right) - \sin(\theta)}{\cos\left(\theta + \frac{\pi}{2}\right) - \cos(\theta)} \\ &= \frac{\cos(\theta) - \sin(\theta)}{-\sin(\theta) - \cos(\theta)} \end{aligned}$$