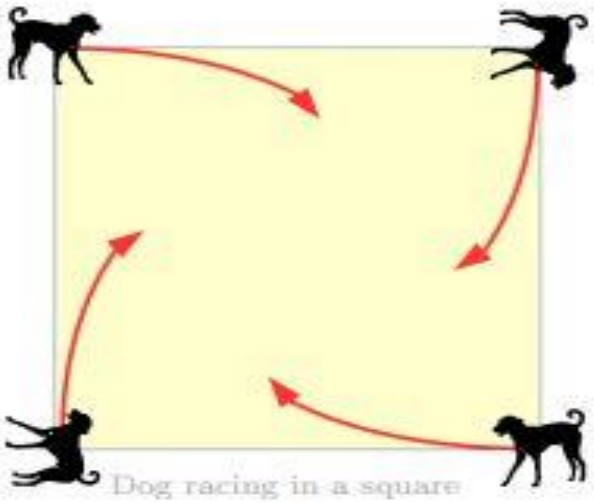
Four dogs are positioned at the corners of a square (d=1m), chase each other in clockwise direction with the same constant speed .

As their target is moving, they will follow a curved path, eventually colliding in the center of the square.



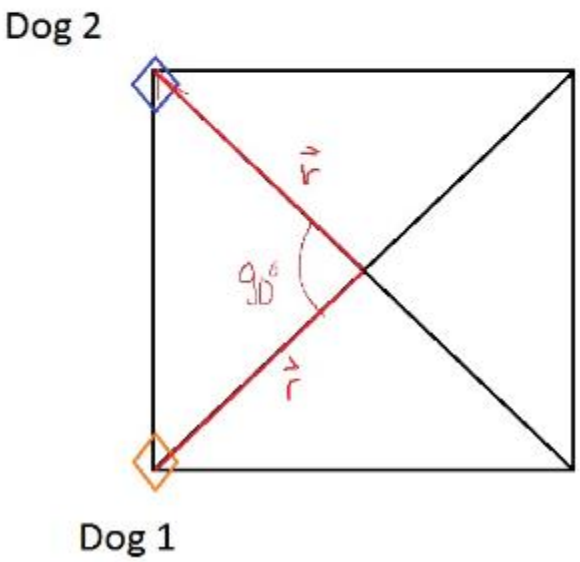

(a) Why is the total length of the path just 1m ?
(b) Find and solve a differential equation for the radius r(θ) in polar coordinates.

**Solution:**

$$r(\theta) = a + b\theta \quad \text{or} \quad r(\theta) = a\theta^{\frac{1}{n}}$$

If *Dog 1* is positioned at $(r, \theta) \implies$ *Dog 2* is positioned at $(r, \theta + \frac{\pi}{2})$

Picture:




$$x_1 = r\cos(\theta)$$
$$y_1 = r\sin(\theta)$$
$$x_2 = r\cos(\theta + \frac{\pi}{2}) = -r\sin(\theta)$$
$$y_2 = r\sin(\theta + \frac{\pi}{2}) = r\cos(\theta)$$

If these are the two position vectors then the vector joining the two points is my velocity vector.

$$\implies \frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{r\sin(\theta + \frac{\pi}{2}) - r\sin(\theta)}{r\cos(\theta + \frac{\pi}{2}) - r\cos(\theta)} = \frac{\sin(\theta + \frac{\pi}{2}) - \sin(\theta)}{\cos(\theta + \frac{\pi}{2}) - \cos(\theta)}$$
$$= \frac{\cos(\theta) - \sin(\theta)}{-\sin(\theta) - \cos(\theta)}$$