

SNS COLLEGE OF TECHNOLOGY



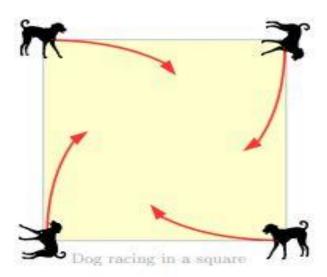
(An Autonomous Institution)
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DEPARTMENT OF MATHEMATICS

UNIT II ORDINARY DIFFERENTIAL EQUATIONS

Four dogs are positioned at the corners of a square (d=1m), chase each other in clockwise direction with the same constant speed.

As their target is moving, they will follow a curved path, eventually colliding in the center of the square.



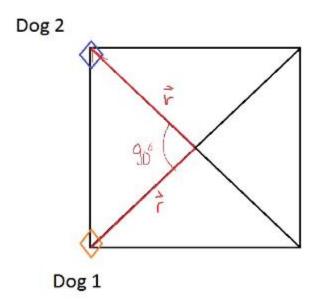
- (a) Why is the total length of the path just 1m?
- (b) Find and solve a differential equation for the radius $r(\theta)$ in polar coordinates.

Solution:

$$r(\theta) = a + b\theta$$
 or $r(\theta) = a\theta^{\frac{1}{n}}$

If Dog 1 is positioned at $(r, \theta) \implies \textit{Dog 2}$ is positioned at $(r, \theta + \frac{\pi}{2})$

Picture:



$$egin{aligned} x_1 &= r\cos(heta) \ y_1 &= r\sin(heta) \ x_2 &= r\cos(heta + rac{\pi}{2}) = -r\sin(heta) \ y_2 &= r\sin(heta + rac{\pi}{2}) = r\cos(heta) \end{aligned}$$

If these are the two position vectors then the vector joining the two points is my velocity vector.

$$\implies \frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{r\sin(\theta + \frac{\pi}{2}) - r\sin(\theta)}{r\cos(\theta + \frac{\pi}{2}) - r\cos(\theta)} = \frac{\sin(\theta + \frac{\pi}{2}) - \sin(\theta)}{\cos(\theta + \frac{\pi}{2}) - \cos(\theta)}$$
$$= \frac{\cos(\theta) - \sin(\theta)}{-\sin(\theta) - \cos(\theta)}$$