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DEPARTMENT OF MATHEMATICS

(14) Find the values of a and b so that the surface $ax^3 - by^2z = (a+3)x^2$ and $4x^2y - z^3 = 11$ may cut orthogonally at (a, -1, -3).

Soln: $a = -7/3 \quad \& b = 64/9$

DIVERGENCE OF A VECTOR POINT FUNCTION:

Let F be any given Continuously differentiable vector point function then the divergence of F is defined as,

$$\operatorname{div} \vec{F} = \nabla \cdot \vec{F} = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}\right) \cdot \vec{F}$$

$$= \vec{i} \cdot \frac{\partial \vec{F}}{\partial x} + \vec{j} \cdot \frac{\partial \vec{F}}{\partial y} + \vec{k} \cdot \frac{\partial \vec{F}}{\partial z}$$

Note :

1. V. F is a scalar point function.

Solenoidal vector: 0 = , W.

Vector F is said to be solenoidal vector if div F = 0.

CURL OF A VECTOR POINT FUNCTION:

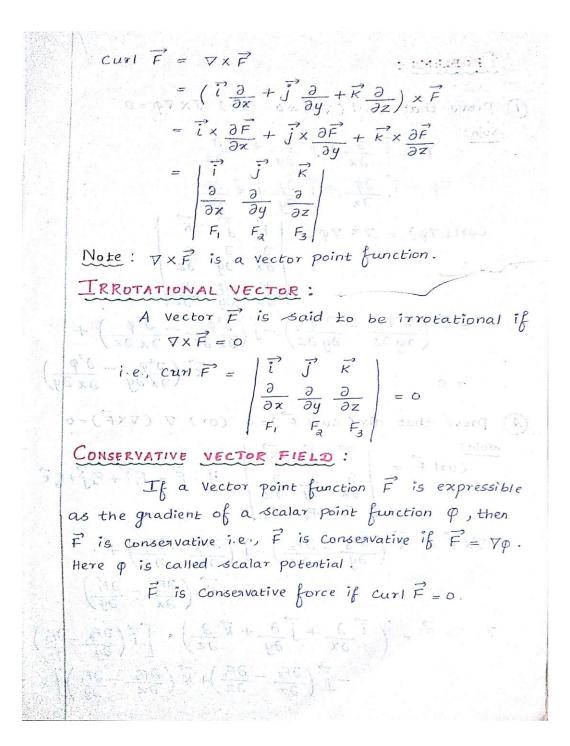
Let $\vec{F} = F_1 \vec{i} + F_2 \vec{j} + F_3 \vec{k}$ be any given continuously differentiable vector point function, the curl or rotation of \vec{F} is defined as,





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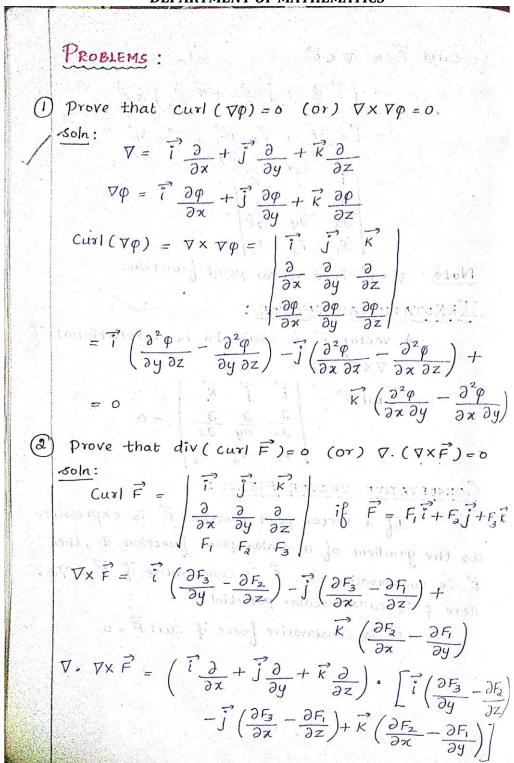
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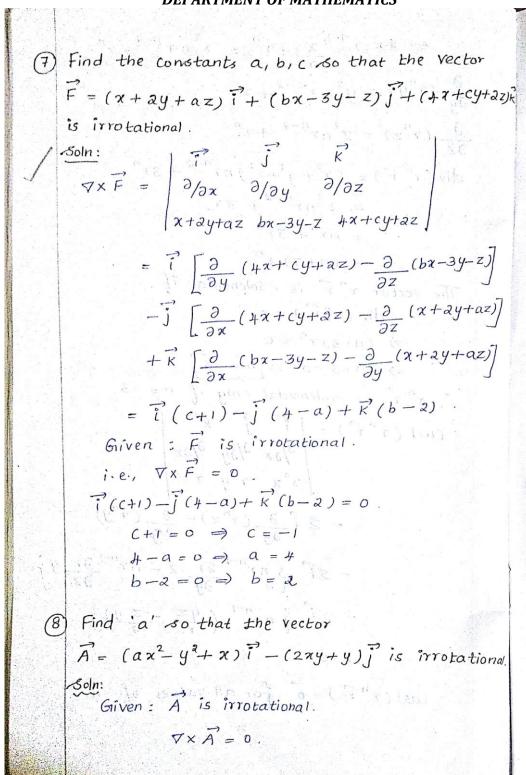
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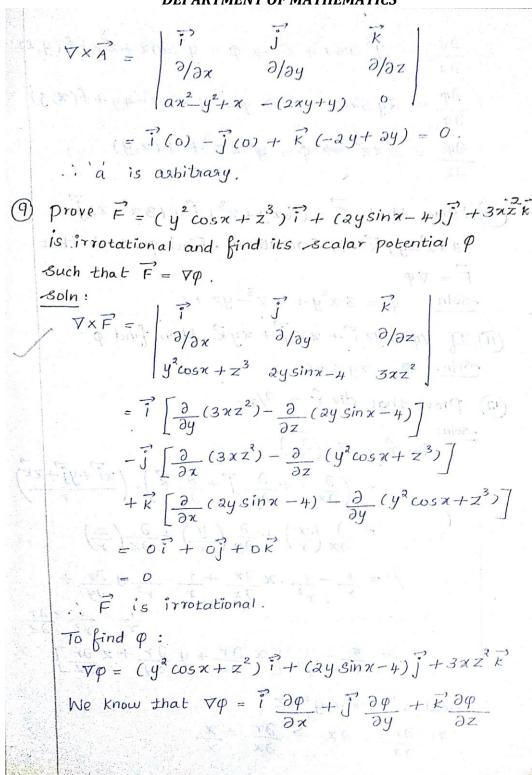




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$$\frac{\partial \varphi}{\partial x} = y^{2} \cos x + z^{3} \Rightarrow \varphi = y^{2} \sin x + z^{3} + f(y,z)$$

$$\frac{\partial \varphi}{\partial y} = \lambda y \sin x - 4 \Rightarrow \varphi = y^{2} \sin x - 4y + f(x,z)$$

$$\frac{\partial \varphi}{\partial z} = 3 \times z^{2} \Rightarrow \varphi = y^{2} \times z^{3} + f(x,y)$$

(3xz²-y) \vec{k} is irrotational. Find φ such that $\vec{F} = \nabla \varphi$.

Soln: $\phi = 3x^2y + xz^3 - yz + c$

- (1) If $\nabla \varphi = yz\vec{i} + \chi z\vec{j} + \chi y\vec{k}$ then find φ .

 Soln: $\varphi = \chi yz + c$
- 12) Prove that $\operatorname{div} \hat{r} = \frac{2}{r}$.

 Soln: $\operatorname{div} \hat{x} = \nabla \cdot \left(\frac{r}{r}\right)$ $= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right) \cdot \left(\frac{x\vec{i} + y\vec{j} + z\vec{k}}{r}\right)$ $= \frac{\partial}{\partial x}\left(\frac{x}{r}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r}\right)$ $= \frac{1}{r} \frac{1}{r^2} \cdot x \cdot \frac{\partial r}{\partial x} + \frac{1}{r} \frac{1}{r^2} \cdot y \cdot \frac{\partial r}{\partial y} + \frac{1}{r^2} \cdot \frac{z}{r^2}$ $= \frac{3}{r} \frac{1}{r^2} \left[x \cdot \frac{\partial r}{\partial x} + y \cdot \frac{\partial r}{\partial y} + z \cdot \frac{\partial r}{\partial z}\right]$

Now $r^2 = x^2 + y^2 + z^2$. $2r \frac{\partial r}{\partial x} = 2x \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}$