



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-I Vector calculus

Gradient and directional derivative

unit-1

Scalar quantities $\frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} + \frac{\partial \phi}{\partial z} \vec{k} = \nabla \phi$

A scalar quantity is that which has only magnitude and it is not related to any direction

Vector quantities (e.g.) $(\sin \theta) \vec{i} + (\cos \theta) \vec{j} + (0) \vec{k}$

A vector quantity is that which has both magnitude and direction

Vector differential operators ∇ is a vector operator

The vector differential operator is denoted by ∇ and it is defined by $\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$

Note $\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$

$\vec{i} \times \vec{j} = \vec{j} \times \vec{k} = \vec{k} \times \vec{i} = 1$

Gradient of a scalar point function

If $\phi(x, y, z)$ is a scalar point function and it is continuously differentiable then it is defined as

$$\nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$
 if it is denoted as
grad (ϕ) or $\nabla \phi$.



Problem 1

Find the gradient $\nabla\phi$, where $\phi = x^2 + y^2 + z^2$

$$\nabla\phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

$$\begin{aligned}\text{using chain rule method, we get,} \\ \text{gradient, } &= \vec{i} \frac{\partial}{\partial x} (x^2 + y^2 + z^2) + \vec{j} \frac{\partial}{\partial y} (x^2 + y^2 + z^2) + \vec{k} \frac{\partial}{\partial z} (x^2 + y^2 + z^2) \\ &= \vec{i}(2x) + \vec{j}(2y) + \vec{k}(2z) \quad (\text{by } \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})\end{aligned}$$

$$\nabla\phi = 2x\vec{i} + 2y\vec{j} + 2z\vec{k} \quad \text{as per formula, we get,}$$

Problem 2

Find $\nabla\phi$ where $\phi = 3x^2y - y^3z^2$, at $(1, 1, 1)$

$$\begin{aligned}\text{Using chain rule method, we get,} \\ \nabla\phi &= \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z} \\ &= \vec{i} \frac{\partial}{\partial x} (3x^2y - y^3z^2) + \vec{j} \frac{\partial}{\partial y} (3x^2y - y^3z^2) + \\ &\quad \vec{k} \frac{\partial}{\partial z} (3x^2y - y^3z^2) \\ &= \vec{i}(6xy) + \vec{j}(3x^2 - 3y^2z^2) + \vec{k}(-2y^3z)\end{aligned}$$

$$\nabla\phi(1, 1, 1) = 6\vec{i} + 0\vec{j} - 2\vec{k}$$

$$\nabla\phi(1, 1, 1) = 6\vec{i} + 0\vec{j} - 2\vec{k}$$

$$\text{by using formula, } \vec{i} \cdot \vec{i} + \vec{j} \cdot \vec{j} + \vec{k} \cdot \vec{k} = \vec{v} \cdot \vec{v} = \phi \nabla\phi$$



Problem 2

Find the directional derivative of $4xz^2 + xy^2$ at the point $(1, -2, 1)$ in the direction $2\vec{i} - \vec{j} - 2\vec{k}$.

The directional derivative is $\frac{\nabla \Phi \cdot \vec{a}}{|\vec{a}|}$

$$\nabla \Phi = \vec{i}(4z^2 + y^2) + \vec{j}(x^2) + \vec{k}(8xz + 2xy)$$

$$\nabla \Phi = 2\vec{i} + \vec{j} + 6\vec{k}$$

$(1, -2, 1)$

$$\frac{2}{\sqrt{4+1+36}} + \frac{-1}{\sqrt{4+1+36}} + \frac{6}{\sqrt{4+1+36}} = \frac{2}{\sqrt{40}} + \frac{-1}{\sqrt{40}} + \frac{6}{\sqrt{40}} = \frac{7}{\sqrt{40}}$$

$$|\vec{a}| = \sqrt{4+1+36}$$

$$|\vec{a}| = 3.$$

$$\therefore \text{the directional derivative is } \frac{(2\vec{i} + \vec{j} + 6\vec{k}) \cdot (2\vec{i} - \vec{j} - 2\vec{k})}{3}$$