



### Divergence of a vector point function

Let  $\vec{F}$  be any given continuously

differentiable point function then the divergence of

$\vec{F}$  is defined as  $\text{div } \vec{F} = \nabla \cdot \vec{F} = \vec{i} \frac{\partial F_1}{\partial x} + \vec{j} \frac{\partial F_2}{\partial y} + \vec{k} \frac{\partial F_3}{\partial z}$

### Solenoidal of a vector

Let a vector  $\vec{F}$  is said to be solenoidal

if  $\nabla \cdot \vec{F} = 0$ .

### Curl of a vector point function

Let  $\vec{F} = F_1 \vec{i} + F_2 \vec{j} + F_3 \vec{k}$  be any given

continuously differentiable vector point function then

the curl or rotation of  $\vec{F}$  is defined by

$$\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

### Irrrotational vector

A vector  $\vec{F}$  is said to be irrotational if

$$\nabla \times \vec{F} = 0$$



# SNS COLLEGE OF TECHNOLOGY

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UNIT-I Vector calculus

Divergence and curl of vector field

Problem 1

Show that  $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$  is solenoidal.

Solenoidal

$$\nabla \cdot \vec{F} = \vec{i} \frac{\partial F_1}{\partial x} + \vec{j} \frac{\partial F_2}{\partial y} + \vec{k} \frac{\partial F_3}{\partial z}$$

$$= \vec{i} \frac{\partial}{\partial x} (3y^4z^2) + \vec{j} \frac{\partial}{\partial y} (4x^3z^2) + \vec{k} \frac{\partial}{\partial z} (-3x^2y^2)$$

$$\nabla \cdot \vec{F} = 0$$

∴ The given  $\vec{F}$  is solenoidal.  $\therefore \nabla \cdot \vec{F} = 0$

Problem 2

Show that  $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$  is irrotational.

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

$$= \vec{i} \left[ \frac{\partial}{\partial y} (xy) - \frac{\partial}{\partial z} (zx) \right] - \vec{j} \left[ \frac{\partial}{\partial x} (xy) - \frac{\partial}{\partial z} (yz) \right] + \vec{k} \left[ \frac{\partial}{\partial x} (zx) - \frac{\partial}{\partial y} (yz) \right]$$

$$= \vec{i} (x - x) - \vec{j} (y - y) + \vec{k} (z - z) = \vec{0}$$

$$\therefore \vec{F} \text{ is irrotational.}$$

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### Problem 3

Find a such that  $(3x-2y+z)\vec{i} + (4x+ay-2)\vec{j} + (x-y+2z)\vec{k}$  is solenoidal.

$$\begin{aligned} \nabla \cdot \vec{F} &= \vec{i} \frac{\partial F_1}{\partial x} + \vec{j} \frac{\partial F_2}{\partial y} + \vec{k} \frac{\partial F_3}{\partial z} \\ &= \vec{i} \frac{\partial}{\partial x} (3x-2y+z) + \vec{j} \frac{\partial}{\partial y} (4x+ay-2) + \vec{k} \frac{\partial}{\partial z} (x-y+2z) \\ \nabla \cdot \vec{F} &= 3\vec{i} + a\vec{j} + 2\vec{k} \end{aligned}$$

$$0 \cdot (\vec{i} + \vec{j} + \vec{k}) = (3\vec{i} + a\vec{j} + 2\vec{k}) \cdot (\vec{i} + \vec{j} + \vec{k})$$

$$0 = 3 + a + 2$$

$$a = -5$$

### Problem 4

Find the constant a, b, c if  $\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$  is irrotational.

$$\begin{aligned} \nabla \times \vec{F} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} \\ &= \vec{i} \left[ \frac{\partial}{\partial y} (F_3) - \frac{\partial}{\partial z} (F_2) \right] - \vec{j} \left[ \frac{\partial}{\partial x} (F_3) - \frac{\partial}{\partial z} (F_1) \right] + \vec{k} \left[ \frac{\partial}{\partial x} (F_2) - \frac{\partial}{\partial y} (F_1) \right] \\ &= \vec{i} \left[ \frac{\partial}{\partial y} (4x+cy+2z) - \frac{\partial}{\partial z} (bx-3y-z) \right] - \vec{j} \left[ \frac{\partial}{\partial x} (4x+cy+2z) - \frac{\partial}{\partial z} (x+2y+az) \right] \\ &\quad + \vec{k} \left[ \frac{\partial}{\partial x} (bx-3y-z) - \frac{\partial}{\partial y} (x+2y+az) \right] \end{aligned}$$



$$0 = \vec{i}^{\wedge} (c+1) - \vec{j}^{\wedge} (4-a) + \vec{k}^{\wedge} (b-2)$$

$$\begin{aligned} c+1 &= 0 \\ c &= -1 \end{aligned}$$

$$\begin{aligned} -4+a &= 0 \\ a &= 4 \end{aligned}$$

$$\begin{aligned} b-2 &= 0 \\ b &= 2, \end{aligned}$$