



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

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Green's Theorem:
Statement:

If R is the closed region of (x, y) bounded by a simple closed curve C , If (M, N) are continuous function of (x, y) having continuous derivatives w.r.t. x & y , i.e., single integral is

$$\int \int_R (M dx + N dy) = \int \int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

Example: $\int \int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$

Verify green theorem for $\int (xy + y^2) dx + x^2 dy$ where x & y is the closed region bounded by $y = x^2$, $y = x$.

Soln:

$$= \int \int_R (x^2 - x^4) dx dy$$



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UNIT-I Vector calculus

Green's theorem

Given, $\int_C (xy + y^2) dx + x dy$

Let $y = x^2$ and $x = y$. The region R is bounded by $x=0$, $x=1$, $y=x^2$, and $y=x$.

Sub $y=x$ in $y=x^2$:
 $x = x^2 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x=0 \cup x=1$

If $x=0$ then $y=0$ | $\frac{\partial N}{\partial x} = 2y + x = 0$ | $\frac{\partial M}{\partial y} = 2x$

If $x=1$ then $y=1$ | $\frac{\partial N}{\partial x} = 2y + x = 3$ | $\frac{\partial M}{\partial y} = 2x$

RHS : $= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$

$= \int_0^1 \int_{x^2}^x (2x - x - 2y) dx dy$

$= \int_0^1 \int_{x^2}^x (x - 2y) dy dx$

$= \int_0^1 \left[\int_{x^2}^x x dy - \int_{x^2}^x 2y dy \right] dx$

$= \int_0^1 \left[(xy) \Big|_{x^2}^x - 2 \left[\frac{y^2}{2} \right] \Big|_{x^2}^x \right] dx$

$= \int_0^1 \left[(x^2 - x^3) - 2 \left[\frac{x^2 - x^4}{2} \right] \right] dx$

$= \int_0^1 \left[(x^2 - x^3) - [x^2 - x^4] \right] dx$

$= \int_0^1 (x^4 - x^3) dx$



$$\begin{aligned}
 &= \int_0^1 x^4 dx - \int_0^1 x^3 dx \\
 &= \left[\frac{x^5}{5} \right]_0^1 - \left[\frac{x^4}{4} \right]_0^1 \\
 &= \frac{1}{5} - \frac{1}{4} = \frac{4-5}{20} = -\frac{1}{20} \\
 \text{LHS : } &\int_C (xy + y^2) dx + x^2 dy = \int_{OA} (xy + y^2) dx + x^2 dy + \\
 &\quad \int_{AO} (xy + y^2) dx + x^2 dy \\
 \text{Along OA, } &x = y, \quad x \in (0, 1) \\
 &\frac{dy}{dx} = 1 \\
 &dy = dx \\
 &= \int_0^1 (x^2 + x^4) dx + x^2 (2x) dx \\
 &= \int_0^1 (x^3 + x^4) dx + 2x^3 dx \\
 &= \int_0^1 (x^3 + x^4) dx + \int_0^1 2x^3 dx \\
 &= \left[\frac{x^4}{4} + \frac{x^5}{5} \right]_0^1 + \left[\frac{2x^4}{4} \right]_0^1 \\
 &= \left[\frac{1}{4} + \frac{1}{5} \right] + \frac{1}{2} \\
 &= \frac{9}{20} + \frac{1}{2} = \frac{19}{20} \\
 \text{Along AO, } &x = 1, \quad y \in (0, 1) \\
 &\frac{dy}{dx} = 0 \\
 &dy = 0
 \end{aligned}$$



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$$\begin{aligned}
 & \int_{AO} (xy + y^2) dx + \int_{OB} x^2 dy = \int_0^b \left[xy + \frac{y^3}{3} \right]_0^b dx = \int_0^b \left(b^2 x + \frac{b^3}{3} \right) dx \\
 & = \int_0^b (x^2 + x^2 + x^2) dx = \int_0^b (3x^2) dx. \quad \text{LHS} \\
 & = \int_0^b 3x^2 dx. \quad \text{RHS} \\
 & = \left[x + \frac{x^3}{3} \right]_0^b = pb^3 + x^3(p + pb) \quad \text{LHS} \\
 & \Rightarrow x + x^3(p + pb) \quad \text{RHS} \\
 & \text{Now, } \\
 & \text{LHS} = \int_{AO} (xy + y^2) dx + \int_{OB} x^2 dy + \int_{AB} (xy + y^2) dx + x^2 dy \\
 & = \frac{19}{20} + (-1) \int_{OB} x^2 dy = pb^3 \\
 & = \frac{19}{20} - 9pb = \frac{-1}{20}(+x + x^2 x) \\
 & \therefore \text{LHS} = \text{RHS} \quad \text{Hence proved.}
 \end{aligned}$$