Second order Linear Differential Equation with constant co-efficient.

Differential equation :

*A differential equation is an equation involving one dependent variable and its derivative with respect to one or more independent variable :

Ordinary differential equation (ODE)

*An ODE is one in which there is only one independent variable and so the derivatives involved in it are ODE.

Linear differential equation :

*A linear differential equation is one in which the dependent variable and its derivatives occur with first degree and there is no product of dependent variable and derivative (or) product of derivative

*A differential equation which is not linear is called as non - linear differential equation

Example :

① $x\left(\frac{d^2y}{dx^2}\right)+y=x^2 \rightarrow$ This is linear

② $y\left(\frac{dy}{dx}\right)+x^2=0 \rightarrow$ This is not linear

Second order linear differential equation with constant coefficient:

* The second linear differential equation is $(a_0 D^2 + a_1 D + a_2) y = R(x)$

To find complimentary function:

* The axillary equation $a_0 m^2 + a_1 m + a_2 = 0$.
* This quadratic equation has two roots say m_1 and m_2

1) If m_1 and m_2 are real and the value is different then

$$C.F = Ae^{m_1 x} + Be^{m_2 x}$$

2) If m_1 and m_2 are real and the roots value are same say $m_1 = m_2 = m$

then, $C.F = (Ax + B) e^{mx}$

3) If m_1 and m_2 are complex number say $m_1 = \alpha + i\beta$, $m_2 = \alpha - i\beta$ then,

$$C.F = e^{\alpha x} [A \cos \beta x + B \sin \beta x]$$

Notes:

1) To find the particular integral $(P.I) = \frac{1}{f(D)} R(x)$

2) $\cosh x = \frac{e^x + e^{-x}}{2}$ 111ly $\sinh x = \frac{e^x - e^{-x}}{2}$

The A.E

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0$$

$$\boxed{m = 2}, \quad \boxed{m = 3}$$

$$C.F = Ae^{2x} + Be^{3x}$$

Example: 2

$$\frac{d^2y}{dx^2} - 6\left(\frac{dy}{dx}\right) + 9y = 0$$

Soln:

$$\frac{d^2}{dx^2} = D^2$$

$$\frac{d^2y}{dx^2} - 6\left(\frac{dy}{dx}\right) + 9y = 0 \qquad \frac{d}{dx} = D$$

$$D^2y - 6Dy + 9y$$

$$(D^2 - 6D + 9)y = 0$$

$$m^2 - 6m + 9 = 0$$

$$m - 3 = 0 \quad | \quad m - 3 = 0$$

$$\boxed{m = 3} \quad \boxed{m = 3}$$

$$C.F = (Ax + B)e^{3x}$$

$Ae^{mx} + Be^{x}$

$(Ax+B)e^{x}$

$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$