Type : II

$R(x) = \cos ax$ (or) $\sin ax$.

* Replace $D^2 = -(a)^2$

$\frac{1}{P}$ = Integrate.

Example: 1.

Solve $(D^2+4)y = \cos 2x$

Soln: Given,

$(D^2+4)y = \cos 2x$

The A.E is $m^2+4=0$

$m^2 = -4$

$m^2 = (-1)4$

$m^2 = i^2 \cdot 2^2$

$m = \pm 2i$

$\alpha+\beta i = 0+2i \Rightarrow \alpha = 0 \; ; \beta = 2$ $\quad a=2$

$C.F = e^{0x}[A\cos 2x + B\sin 2x]$ $\quad D^2 = -(a)^2 = -4$

$P.I = \frac{1}{D^2+4}\cos 2x$

$P.I = \frac{1}{0}\cdot \cos 2x.$

$P.I = \frac{x}{2D}\cdot \cos 2x$

$= \frac{x}{2}\int \cos 2x \cdot dx$

$= \frac{x}{2}\cdot \frac{\sin 2x}{2}$

$P.I = \frac{x \sin 2x}{4}$

$y = C.F + P.I$

Type : (iii)

$R(x) = x^n$

1. $(1+x)^{-1} = 1 - x + x^2 + \cdots$
2. $(1-x)^{-1} = 1 + x + x^2 + \cdots$
3. $(1+x)^{1} = 1 + x + x^2 + \cdots$
4. $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$
5. $\frac{1}{D} = \int f(x) \cdot dx$
6. D = differentiate

Example: 1

Find the particular integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$

Soln: $D^2y + Dy = x^2 + 2x + 4$

$(D^2 + D)y = x^2 + 2x + 4$

The A.E is $m^2 + m = 0$

$m(m+1) = 0$

$m = 0 \quad m = -1$

$C.F = Ae^{0x} + Be^{-x}$

$$P.I = \frac{1}{f(D)} \cdot R(x)$$

$$= \frac{1}{D(D+1)} \cdot x^2 + 2x + 4$$

$$= \frac{1}{D(D+1)} x^2 + 2x + 4$$

$$= \frac{1}{D}(D+1)^{-1}(x^2 + 2x + 4)$$

$$= \frac{1}{D}\left[1 - D + D^2 + \cdots\right](x^2 + 2x + 4)$$

$$= \frac{1}{D}\left[x^2+2x+4-D(x^2+2x+4)+D^2(x^2-2x+4)\ldots\right]$$

$$= \frac{1}{D}\left[x^2+2x+4-2x-2+2\right]$$

$$= \frac{1}{D}(x^2+4)$$

$$= \int (x^2+4)\,dx$$

$$P.I = \frac{x^3}{3}+4x.$$

$$y = C.F + P.I$$

$$y = (Ae^{0x}+Be^{-x}) + \frac{x^3}{3}+4x.$$