



## SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



$$\therefore \int_C M dx + N dy = \int_{OA} M dx + N dy + \int_{AO} M dx + N dy$$

$$= -1 + \frac{5}{2}$$

$$= \frac{-2+5}{2}$$

$$\boxed{\int_C M dx + N dy = \frac{3}{2}} \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2}$$

Hence Green's theorem is Verified.

### Gauss Divergence theorem:

If  $\vec{F}$  is a vector point function, finite and differentiable in a region  $R$  bounded by a closed surface  $S$ , then the surface integral of the normal component of  $\vec{F}$  taken over  $S$  is equal to the integral of divergence of  $\vec{F}$  taken over  $V$ .

$$\boxed{\iint_S \vec{F} \cdot \hat{n} ds = \iiint_V \nabla \cdot \vec{F} dv}$$

Where  $\hat{n}$  is the unit vector in the positive (outward drawn) normal to  $S$ .

### Problems:-

1) Verify Gauss divergence theorem for  $\vec{F} = 4xz\vec{i} - y\vec{j} + yz\vec{k}$

Over the cube  $x=0, x=1, y=0, y=1, z=0, z=1$ .



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Sol.:

By Gauss divergence theorem,

$$\iint_S \vec{F} \cdot \hat{n} \, ds = \iiint_V \nabla \cdot \vec{F} \, dv$$

RHS:

$$\nabla \cdot \vec{F} = \left( \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) \cdot (4xz \vec{i} - y^2 \vec{j} + yz \vec{k})$$

$$= \frac{\partial}{\partial x} (4xz) + \frac{\partial}{\partial y} (-y^2) + \frac{\partial}{\partial z} (yz)$$

$$= 4z - 2y + y$$

$$= 4z - y$$

$$\boxed{\nabla \cdot \vec{F} = 4z - y}$$

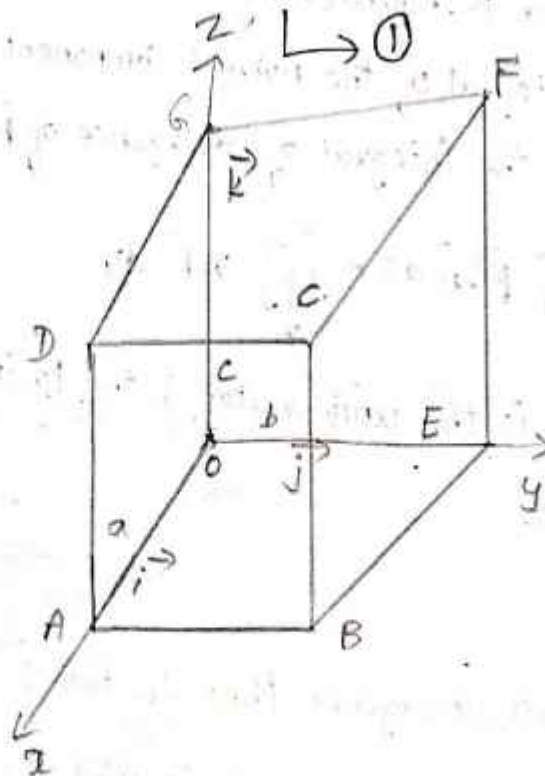
$$\iiint_V \nabla \cdot \vec{F} \, dv = \int_0^1 \int_0^1 \int_0^1 (4z - y) \, dx \, dy \, dz = \frac{3}{2}$$

L.H.S:

$$\iint_S \vec{F} \cdot \hat{n} \, ds$$

$$= \iint_{S_1} + \iint_{S_2} + \iint_{S_3} +$$

$$\iint_{S_4} + \iint_{S_5} + \iint_{S_6}$$





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Surface	$\hat{n}$	$ds$	Face equation
$S_1 - ABCD$	$\hat{i}$	$dydz$	$x=1$
$S_2 - O EFG$	$-\hat{i}$	$dydz$	$x=0$
$S_3 - B C E F$	$\hat{j}$	$dx dz$	$y=1$
$S_4 - O A D G$	$-\hat{j}$	$dx dz$	$y=0$
$S_5 - D C G F$	$\hat{k}$	$dx dy$	$z=1$
$S_6 - O A B E$	$-\hat{k}$	$dx dy$	$z=0$

$$\iint_{S_1} \vec{F} \cdot \hat{n} ds = \iint_{ABCD} (4xz\hat{i} - y^2\hat{j} + yz\hat{k}) \cdot \hat{i} dy dz$$

$$= \int_0^1 \int_0^1 4xz dy dz$$

$$= \int_0^1 \int_0^1 4z dy dz \quad (\because x=1)$$

$$= 2$$

$$\boxed{\iint_{S_1} \vec{F} \cdot \hat{n} ds = 2}$$

$$\iint_{S_2} \vec{F} \cdot \hat{n} ds = \iint_{O EFG} (4xz\hat{i} - y^2\hat{j} + yz\hat{k}) \cdot (-\hat{i}) dy dz$$

$$= \int_0^1 \int_0^1 (-4xz) dy dz \quad (\because x=0)$$

$$= 0$$

$$\boxed{\iint_{S_2} \vec{F} \cdot \hat{n} ds = 0}$$



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$$\iint_{S_3} \vec{F} \cdot \hat{n} ds = \iint_{BCFE} (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \cdot \vec{j} dx dz$$

$$= \int_0^1 \int_0^1 (-y) dx dz \quad (\text{Here } y=1)$$

$$= \int_0^1 \int_0^1 -dx dz$$

$$= -1$$

$$\boxed{\iint_{S_3} \vec{F} \cdot \hat{n} ds = -1}$$

$$\iint_{S_4} \vec{F} \cdot \hat{n} ds = \iint_{OADC} (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \cdot (-\vec{j}) dx dz$$

$$= \int_0^1 \int_0^1 y^2 dx dz = 0 \quad (\because y=0)$$

$$\boxed{\iint_{S_4} \vec{F} \cdot \hat{n} ds = 0}$$

$$\iint_{S_5} \vec{F} \cdot \hat{n} ds = \iint_{DCGF} (4xz\vec{i} - y^2\vec{j} + yz\vec{k}) \cdot \vec{k} dx dy$$

$$= \int_0^1 \int_0^1 yz dx dy$$

$$= \int_0^1 \int_0^1 y dx dy \quad (\because z=1)$$





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$$= \int_0^1 y [x]_0^1 dy$$

$$= \int_0^1 y dy$$

$$= \left[ \frac{y^2}{2} \right]_0^1$$

$$= \frac{1}{2}$$

$$\boxed{\iint_{S_5} \vec{F} \cdot \hat{n} ds = \frac{1}{2}}$$

$$\iint_{S_6} \vec{F} \cdot \hat{n} ds = \iint_{OABE} (-yz) dx dy = 0 \quad (\because z=0)$$

$$\boxed{\iint_{S_6} \vec{F} \cdot \hat{n} ds = 0}$$

$$\iint_S \vec{F} \cdot \hat{n} ds = \iint_{S_1} + \iint_{S_2} + \iint_{S_3} + \iint_{S_4} + \iint_{S_5} + \iint_{S_6}$$

$$= 2 + 0 + (-1) + 0 + \frac{1}{2} + 0$$

$$= 2 - 1 + \frac{1}{2}$$

$$= 1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2} \rightarrow \textcircled{2}$$

$$\boxed{\iint_S \vec{F} \cdot \hat{n} ds = \frac{3}{2}}$$

From ① & ②,

$$\boxed{\iint_S \vec{F} \cdot \hat{n} ds = \iiint_V \text{div} \vec{F} dv}$$



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2) Verify divergence theorem for  $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$  taken over the rectangle Parallelepiped  $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$ .

(or)

Verify Gauss divergence theorem for  $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$  and  $S$  is the surface of the rectangular Parallelepiped bounded by  $x=0, x=a, y=0, y=b, z=0, z=c$ .

Sol: By Gauss-divergence theorem,

$$\iint_S \vec{F} \cdot \hat{n} \, ds = \iiint_V \text{div} \vec{F} \, dv$$

RHS:

$$\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$$

$$\text{div} \vec{F} = \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot \left( (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k} \right)$$

$$= \frac{\partial}{\partial x} (x^2 - yz) + \frac{\partial}{\partial y} (y^2 - zx) + \frac{\partial}{\partial z} (z^2 - xy)$$

$$= 2x + 2y + 2z$$

$$= 2(x + y + z)$$

$$\boxed{\text{div} \vec{F} = 2(x + y + z)}$$



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$$\iiint_V \nabla \cdot \vec{F} \, dV = \int_0^a \int_0^b \int_0^c 2(x+y+z) \, dx \, dy \, dz$$

$$= 2 \int_0^a \int_0^b \left[ xz + yz + \frac{z^2}{2} \right]_0^c \, dx \, dy$$

$$= 2 \int_0^a \int_0^b \left( xc + yc + \frac{c^2}{2} \right) \, dx \, dy$$

$$= 2 \int_0^a \left[ xyc + \frac{y^2}{2} c + \frac{c^2}{2} y \right]_0^b \, dx$$

$$= 2 \int_0^a \left( xbc + \frac{b^2 c}{2} + \frac{c^2 b}{2} \right) \, dx$$

$$= 2 \left[ \frac{x^2}{2} bc + \frac{b^2 c}{2} x + \frac{c^2 b}{2} x \right]_0^a$$

$$= 2 \left[ \frac{a^2 bc}{2} + \frac{abc^2}{2} + \frac{abc^2}{2} \right]$$

$$= abc(a+b+c) \rightarrow \textcircled{1}$$

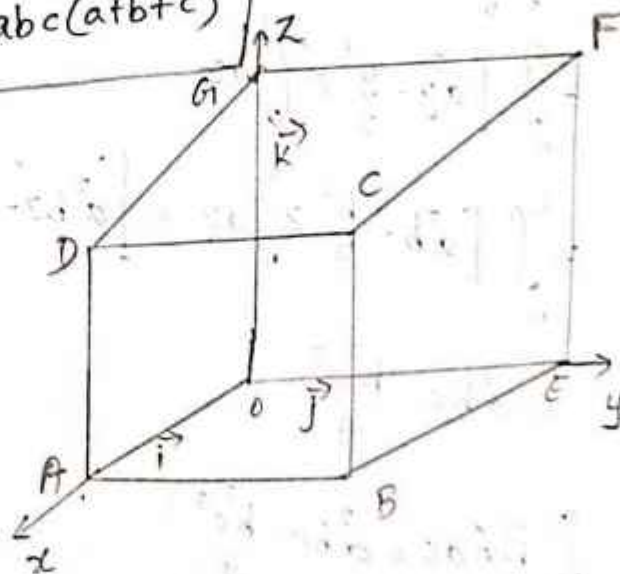
$$\boxed{\iiint_V \nabla \cdot \vec{F} \, dV = abc(a+b+c)}$$

LHS :

$$\iint_S \vec{F} \cdot \hat{n} \, ds$$

$$= \iint_{S_1} + \iint_{S_2} + \iint_{S_3}$$

$$+ \iint_{S_4} + \iint_{S_5} + \iint_{S_6}$$





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Surface	$\hat{n}$	ds	face equation
$S_1$ - ABCD	$\hat{i}$	dydz	$x=a$
$S_2$ - O EFG	$-\hat{i}$	dydz	$x=0$
$S_3$ - B C E F	$\hat{j}$	dx dz	$y=b$
$S_4$ - O A D G	$-\hat{j}$	dx dz	$y=0$
$S_5$ - D C G F	$\hat{k}$	dx dy	$z=c$
$S_6$ - O A B E	$-\hat{k}$	dx dy	$z=0$

$$\iint_{S_1} \vec{F} \cdot \hat{n} ds = \iint_{ABCD} \{ [x^2 - yz] \hat{i} + [y^2 - zx] \hat{j} + [z^2 - xy] \hat{k} \} \cdot \hat{i} dy dz$$

$$= \int_0^c \int_0^b (x^2 - yz) dy dz$$

$$= \int_0^c \int_0^b (a^2 - yz) dy dz \quad ( \because x = a )$$

$$= \int_0^c \left[ ay - \frac{y^2}{2} z \right]_0^b dz$$

$$= \int_0^c \left[ a^2 b - \frac{b^2}{2} z \right] dz = \left[ a^2 bz - \frac{b^2}{2} \frac{z^2}{2} \right]_0^c$$

$$= a^2 bc - \frac{b^2 c^2}{4}$$

$$\boxed{\iint_{S_1} \vec{F} \cdot \hat{n} ds = a^2 bc - \frac{b^2 c^2}{4}}$$





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$$\iint_{S_2} \vec{F} \cdot \hat{n} \, ds = \iint_{OEFH} [(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}] \cdot (-\vec{i}) \, dy \, dz$$

$$= \int_0^c \int_0^b -(x^2 - yz) \, dy \, dz$$

$$= \int_0^c \int_0^b yz \, dy \, dz \quad (\because x=0)$$

$$= \int_0^c \left[ \frac{y^2}{2} \right]_0^b z \, dz$$

$$= \frac{b^2}{2} \left[ \frac{z^2}{2} \right]_0^c$$

$$= \frac{b^2 c^2}{4}$$

$$\boxed{\iint_{S_2} \vec{F} \cdot \hat{n} \, ds = \frac{b^2 c^2}{4}}$$

$$\iint_{S_3} \vec{F} \cdot \hat{n} \, ds = \iint_{BCEF} [(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}] \cdot \vec{j} \, dx \, dz$$

$$= \int_0^a \int_0^c (y^2 - zx) \, dx \, dz$$

$$= \int_0^a \int_0^c (b^2 - zx) \, dx \, dz \quad (\because y=b)$$

$$= \int_0^a \left[ b^2 x - \frac{zx^2}{2} \right]_0^c \, dz$$

$$= \int_0^a \left[ b^2 c - \frac{zc^2}{2} \right] \, dz$$



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$$= \left[ b^2 cz - \frac{z^2}{4} c^2 \right]_0^a$$

$$= abc^2 - \frac{ac^2}{4}$$

$$\boxed{\iint_{S_3} \vec{F} \cdot \hat{n} ds = abc^2 - \frac{ac^2}{4}}$$

$$\iint_{S_4} \vec{F} \cdot \hat{n} ds = \iint_{OADG} [(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}] (-\vec{j}) dx dz$$

$$= \int_0^a \int_0^c -(y^2 - zx) dx dz$$

$$= \int_0^a \int_0^c zx dx dz \quad (\because y=0)$$

$$= \int_0^a \left[ \frac{x^2}{2} \right]_0^c z dz$$

$$= \frac{c^2}{2} \left[ \frac{z^2}{2} \right]_0^a$$

$$= \frac{ac^2}{2}$$

$$\boxed{\iint_{S_4} \vec{F} \cdot \hat{n} ds = \frac{ac^2}{2}}$$

$$\iint_{S_5} \vec{F} \cdot \hat{n} ds = \iint_{DCGF} [(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}] \vec{k} dx dy$$



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$$= \int_0^a \int_0^b (z^2 - xy) dx dy$$

( $\because z = c$ )

$$= \int_0^a \int_0^b (c^2 - xy) dx dy$$

$$= \int_0^a \left[ cx - \frac{x^2}{2} y \right]_0^b dy$$

$$= \int_0^a \left( bc^2 - \frac{by^2}{2} \right) dy$$

$$= \left( bcy - \frac{by^3}{2} \right)_0^a$$

$$= abc^2 - \frac{ab^2}{4}$$

$$\boxed{\iint_{S_5} \vec{F} \cdot \hat{n} ds = abc^2 - \frac{ab^2}{4}}$$

$$\iint_{S_6} \vec{F} \cdot \hat{n} ds = \iint_{OABE} [(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}] \cdot (-\vec{k}) dx dy$$

$$= \int_0^a \int_0^b -(z^2 - xy) dx dy$$

( $\because z = 0$ )

$$= \int_0^a \int_0^b xy dx dy$$

$$= \int_0^a y dy \left[ \frac{x^2}{2} \right]_0^b = \frac{b^2}{2} \left[ \frac{y^2}{2} \right]_0^a = \frac{ab^2}{4}$$

$$\boxed{\iint_{S_6} \vec{F} \cdot \hat{n} ds = \frac{ab^2}{4}}$$



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$$\iint_S \vec{F} \cdot \hat{n} \, ds = abc - \frac{bc^2}{4} + \frac{b^2c}{4} + abc - \frac{a^2c}{4} + \frac{ac^2}{4} + ab$$

$$- \frac{ab^2}{4} + \frac{a^2b}{4}$$

$$= abc(a+b+c) \rightarrow (2)$$

$$\boxed{\iint_S \vec{F} \cdot \hat{n} \, ds = abc(a+b+c)}$$

From (1) and (2),

$$\iint_S \vec{F} \cdot \hat{n} \, ds = \iiint_V \nabla \cdot \vec{F} \, dv$$

3) Verify Gauss divergence theorem for the function

$\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$  over the cylindrical region bounded by

$x^2 + y^2 = 9$ ,  $z = 0$  and  $z = 2$ .

Sol.: By Gauss divergence theorem,

$$\iint_S \vec{F} \cdot \hat{n} \, ds = \iiint_V \nabla \cdot \vec{F} \, dv$$

RHS:

$$\text{Given: } \vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$$

$$\nabla \cdot \vec{F} = \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (y\vec{i} + x\vec{j} + z^2\vec{k})$$

$$= \frac{\partial}{\partial x} (y) + \frac{\partial}{\partial y} (x) + \frac{\partial}{\partial z} (z^2)$$

$$= 0 + 0 + 2z$$

