



Conditional probability :

The conditional probability of an event B assuming that the event A has happened, is defined as,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ provided } P(A) \neq 0$$

Similarly,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ provided } P(B) \neq 0$$

Theorem :

If A and B are independent, then prove that

1. \bar{A} and B are independent
2. A and \bar{B} are independent
3. \bar{A} and \bar{B} are independent.

Proof: Given A and B are independent $\Rightarrow P(A \cap B) = P(A)P(B)$

1. $P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B)$ → (1)

Consider,

$$B = (A \cap B) \cup (\bar{A} \cap B)$$

$$P(B) = P[(A \cap B) \cup (\bar{A} \cap B)]$$

$$P(B) = P(A \cap B) + P(\bar{A} \cap B)$$

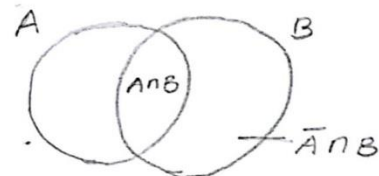
$$\Rightarrow P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$= P(B) - P(A)P(B) \quad (\text{from (1)})$$

$$= P(B) [1 - P(A)]$$

$$\boxed{P(\bar{A} \cap B) = P(B) \cdot P(\bar{A})}$$

$\therefore \bar{A}$ and B are independent.





④

$$2. \underline{P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})}$$

Consider,

$$A = (A \cap \bar{B}) \cup (A \cap B)$$

$$P(A) = P[(A \cap \bar{B}) \cup (A \cap B)]$$

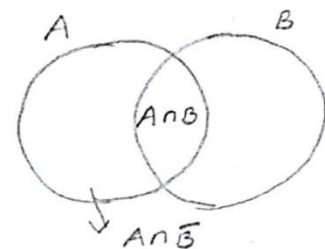
$$= P(A \cap \bar{B}) + P(A \cap B)$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$= P(A) - P(A)P(B)$$

$$= P(A) [1 - P(B)]$$

$$\boxed{P(A \cap \bar{B}) = P(A) P(\bar{B})} \quad \therefore A \text{ and } \bar{B} \text{ are independent}$$



$$3. \underline{P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})}$$

Consider,

$$P(\bar{A} \cap \bar{B}) = \overline{P(A \cup B)} \quad (\text{by De Morgan's law})$$

$$= 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - [P(A) + P(B) - P(A) \cdot P(B)]$$

$$= 1 - P(A) - P(B) + P(A) \cdot P(B)$$

$$= P(\bar{A}) - P(B) [1 - P(A)]$$

$$= P(\bar{A}) - P(B) \cdot P(\bar{A})$$

$$= P(\bar{A}) [1 - P(B)]$$

$$\boxed{P(\bar{A} \cap \bar{B}) = P(\bar{A}) P(\bar{B})}$$

$\therefore \bar{A}$ and \bar{B} are independent.





PROBLEMS :

- ① From a bag containing 5 white balls and 6 green balls, 3 balls are drawn with replacement. What is the chance that (i) all are of same colour (ii) they are alternatively of different colours.

Solution:

$$S = \{ 5W, 6G \}$$

(i) $P(\text{all are of same colour})$

$$= P(\text{all are white or all are green})$$

$$= P(\text{all are white}) + P(\text{all are green})$$

$$= P(IW \ IIW \ IIIW) + P(IG \ IIG \ IIIG)$$

$$= \frac{5}{11} \times \frac{5}{11} \times \frac{5}{11} + \frac{6}{11} \times \frac{6}{11} \times \frac{6}{11}$$

$$= \frac{125}{1331} + \frac{216}{1331}$$

$$= \frac{341}{1331}$$

(ii) $P(\text{they are alternatively of different colours})$

$$= P(IW \ IIG \ IIIG \ \text{or} \ \ IIG \ IIW \ IIIG)$$

$$= \frac{5}{11} \times \frac{6}{11} \times \frac{5}{11} + \frac{6}{11} \times \frac{5}{11} \times \frac{6}{11}$$

$$= \frac{150}{1331} + \frac{180}{1331}$$

$$= \frac{330}{1331}$$



② If A and B are events with $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, find $P(A^c \cap B^c)$.

Soln:

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{3}{8} + \frac{1}{2} - \frac{1}{4} = \frac{5}{8} \end{aligned}$$

$$\begin{aligned} P(A^c \cap B^c) &= P[(A \cup B)^c] \\ &= 1 - P(A \cup B) \\ &= 1 - \frac{5}{8} = \frac{3}{8} \end{aligned}$$

③ If $P(A) = 0.4$, $P(B) = 0.7$, $P(A \cap B) = 0.3$, find $P(\bar{A} \cap \bar{B})$ & $P(\bar{A} \cup \bar{B})$.

Soln:

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.4 + 0.7 - 0.3 \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} P(\bar{A} \cap \bar{B}) &= P(\overline{A \cup B}) \\ &= 1 - P(A \cup B) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} P(\bar{A} \cup \bar{B}) &= P(\overline{A \cap B}) \\ &= 1 - P(A \cap B) \\ &= 1 - 0.3 \\ &= 0.7 \end{aligned}$$