Binomial Distribution:

Bernoulli trial:

Each trial has two possible outcomes generally called Success (p) and failure (q). Such a trial is known as Bernoulli trial.

Binomial Distribution:

A random variable x is said to follow Binomial distribution, if it assume only non-negative values of its probability mass function is given by,

$$P[X=x] = nC_x\, p^x q^{n-x}, \quad x = 0,1,\ldots,n \text{ and } p+q=1$$

Properties:

i). There must be a fixed number of trials.

ii). All trials must have identical probabilities of Success (P)

iii). The trials must be independent of each other.

MGF, mean and variance of Binomial Distribution:

$$M_X(t) = \sum_{x=0}^{\infty} e^{tx} p(x)$$

$$= \sum_{x=0}^{n} e^{tx}\, nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} (pe^t)^x\, nC_x\, q^{n-x}$$

$$= nC_0 q^n + nC_1 (pe^t)^1 q^{n-1} + nC_2 (pe^t)^2 q^{n-2} + \ldots + nC_n (pe^t)^n$$

$$= q^n + nc_1(pe^t)q^{n-1} + nc_2(pe^t)^2 q^{n-2} + \dots + (pe^t)^n$$

$$\text{MGF} = (q+pe^t)^n \qquad \boxed{\because (q+p)^n = q^n + nc_1 q^{n-1} p + \dots + p^n}$$

Mean:

$$E[X] = \left[\frac{d}{dt} M_X(t)\right]_{t=0}$$

$$= \left[n(q+pe^t)^{n-1} pe^t\right]_{t=0}$$

$$= np(q+p)^{n-1}$$

$$= np\, 1^{n-1} \qquad \because p+q=1$$

mean $= np$

Variance:

$$Var(X) = E[X^2] - (E[X])^2$$

$$E[X^2] = \left[\frac{d^2}{dt^2} M_X(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[\frac{d}{dt} M_X(t)\right]_{t=0}$$

$$= \left[\frac{d}{dt}\left[np(q+pe^t)^{n-1} e^t\right]\right]_{t=0}$$

$$= np\left[(q+pe^t)^{n-1} e^t + e^t (n-1)(q+pe^t)^{n-2} pe^t\right]_{t=0}$$

$$\because d(uv) = uv' + vu'$$

$$= np\left[(q+p)^{n-1} + (n-1)p(q+p)^{n-2}\right]$$

$$= np\left[1^{n-1} + (n-1)p\, 1^{n-2}\right] \qquad \because p+q=1$$

$$= np\left[1+(n-1)p\right]$$

$$= np\,[1+np-p]$$

$$E[x^2] = np+n^2p^2-np^2$$

$$Var[x] = E(x^2) - [E(x)]^2$$

$$= np+n^2p^2-np^2-n^2p^2$$

$$= np-np^2$$

$$= np(1-p)$$

$$Var(x) = npq$$

$\therefore$ $$M_x(t) = (q+pe^t)^n$$

mean $= np$

Variance $= npq$

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1. For a Binomial variate, mean is 36, variance is 12. Find p, q, n.

Soln.

Given mean: $np = 36 \rightarrow (1)$

variance: $npq = 12 \rightarrow (2)$

$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{12}{36}$$

$$\boxed{q = \frac{1}{3}}$$

WKT $p+q=1$

$$p+\frac{1}{3}=1$$

$$p = 1-\frac{1}{3} = \frac{3-1}{3}$$

$$\boxed{p = \frac{2}{3}}$$

$$(1) \Rightarrow n\left(\frac{2}{3}\right) = 36 \Rightarrow n = 36\left(\frac{3}{2}\right)$$

$$\boxed{n = 54}$$

2]. Find the Binomial distribution for which mean is 4, variance is 3.

Soln.

Given mean: $np = 4 \rightarrow (1)$

variance: $npq = 3 \rightarrow (2)$

$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{3}{4}$$

$$q = \frac{3}{4}$$

WKT $p+q = 1$

$$P = 1 - \frac{3}{4}$$

$$P = \frac{1}{4}$$

$$(1) \Rightarrow n\left(\frac{1}{4}\right) = 4$$

$$n = 16$$

Binomial distribution is

$$P[X=x] = nC_x \, p^x q^{n-x}$$

$$P[X=x] = 16C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{16-x}$$

3]. The mean of BD is 20 and Standard deviation is 4. Determine the parameter of distribution. (n, p, q)

Soln.:

Given mean: $np = 20 \rightarrow (1)$

SD : $\sqrt{\text{variance}} \Rightarrow npq = 4^2 = 16 \rightarrow (2)$

$\frac{(2)}{(1)} \Rightarrow \quad \frac{npq}{np} = \frac{16}{20}$

$q = \frac{4}{5}$

$\therefore \quad p+q = 1$

$p = 1 - \frac{4}{5}$

$p = \frac{1}{5}$

$(1) \Rightarrow \quad n\left(\frac{1}{5}\right) = 20$

$n = 100$

4]. If x is a Binomial variate with $n=6$ and $9P[x=4] = P[x=2]$. Find Binomial distribution.

Soln.

Given $9P(x=4) = P(x=2)$

BD: $P[x=x] = nC_x \, p^x q^{n-x}$

$9[6C_4 \, p^4 q^{6-4}] = 6C_2 \, p^2 q^{6-2}$

$9[\cancel{6C_2} \, p^4 q^2] = \cancel{6C_2} \, p^2 q^4$

$9 \frac{p^4}{p^2} = \frac{q^4}{q^2}$

$9p^2 = q^2$

$= (1-p)^2$

$9p^2 = 1 + p^2 - 2p$

$9p^2 - p^2 + 2p - 1 = 0$

$8p^2 + 2p - 1 = 0$

$8p^2 + 4p - 2p - 1 = 0$

-8

$4 \quad -2 = 2$

$$4P(2P+1)-1(2P+1)=0$$
$$(2P+1)(4P-1)=0$$
$$2P+1=0 \quad \Big| \quad 4P-1=0$$
$$P=-1/2 \quad \Big| \quad P=1/4$$

$\therefore P = 1/4$ ($\because$ negative value is not possible)

$\therefore P+q=1$

$$q = 1 - \frac{1}{4}$$
$$q = \frac{3}{4}$$

BD:

$$P[X=x] = 6C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{6-x}$$

5]. ~~Four~~ n coins are tossed simultaneously. What is the probability of getting

i). 2 heads ii). atleast 2 heads

iii). atmost 2 heads.

Soln.

Given: 4 coins tossed. So $n=4$

probability of getting head $p = \frac{1}{2}$

$\therefore q = \frac{1}{2}$

BD:

$$P[X=x] = nC_x\, p^x q^{n-x}$$
$$= 4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$$

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i). 2 heads

$P[X=2] = 4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2}$

$= \frac{4\times3}{2\times1} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

$= 6\left(\frac{1}{2}\right)^4 = \frac{6}{16}$

$= 0.375$

ii). atleast 2 heads

$P[X \geq 2] = P(X=2) + P(X=3) + P(X=4)$

$= 4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-2} + 4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{4-3} + 4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{4-4}$

$= \frac{4\times3}{2\times1}\left(\frac{1}{2}\right)^4 + 4C_1\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4$

$= \left(\frac{1}{2}\right)^4 [6+4+1]$

$= \frac{11}{16}$

$= 0.688$

iii). atmost 2 heads

$P[X \leq 2] = P(X=0) + P(X=1) + P(X=2)$

$= 4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{4-0} + 4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{4-1} + 4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-2}$

$= \left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^4 + \frac{4\times3}{2\times1}\left(\frac{1}{2}\right)^4$ $\boxed{\because nC_0 = 1,\ nC_1 = n}$

$$= \left(\frac{1}{2}\right)^4 [1+4+6]$$

$$= \frac{11}{16}$$

$P[X \le 2] = 0.688$

$P[X \le 2]$

$= 1 - P[X > 2]$

$= 1 - [P(X=3) + P($

6]. 6 dice are thrown 729 times. How many times do you except atleast 3 dice to show 5 or 6.

Soln.

Give $n = 6$

$N = 729$

BD: $P[X=x] = nC_x \, p^x q^{n-x}$

Probability of getting 5 or 6 in the dice.

$= P(5) + P(6)$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

$p = \frac{1}{3}$

$\therefore p+q = 1$

$q = 1 - \frac{1}{3}$

$q = \frac{2}{3}$

atleast 3 dice

$P[X \geq 3] = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= 6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{6-3} + 6C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^{6-4} +$

$6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{6-5} + 6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{6-6}$

$= \frac{\cancel{6}^{2} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 + 6C_2 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + 6C_1 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)$

$+ \left(\frac{1}{3}\right)^6 (1)$

$= 20 \frac{1}{27} \left(\frac{8}{27}\right) + \frac{\cancel{6}^{3} \times 5}{\cancel{2} \times 1} \left(\frac{1}{81}\right) \left(\frac{4}{9}\right) + 6 \left(\frac{1}{243}\right) \left(\frac{2}{3}\right) + \frac{1}{729}$

$= 0.219 + 0.082 + 0.016 + 0.001$

$= 0.318$

For single time, possibility is 0.318

For 729 times, 729×0.318

$= 231.82$

$= 232.$

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7). In a large consignment of electric bulbs 20% are defective. A random sample of 20 is taken for inspection. Find the probability that

i). All are good

ii). Atmost there are 3 defectives.

Soln.:

$n = 20$

$p = 10\%$ defective $= \frac{10}{100}$

$p = 0.1$

$\therefore p + q = 1$

$q = 1 - p = 1 - 0.1$

$q = 0.9$

BD: $P[X = x] = nC_x \, p^x \, q^{n-x}$

$= 20C_x (0.1)^x (0.9)^{20-x}$

i). All are good (no one is defective)

$P[X=0] = 20C_0 (0.1)^0 (0.9)^{20-0}$

$= 1(0.9)^{20}$

$= 0.122$

ii). Atmost there are 3 defectives.

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.122 + 20C_1 (0.1)(0.9)^{20-1} + 20C_2 (0.1)^2 (0.9)^{20-2}$

$+ 20C_3 (0.1)^3 (0.9)^{20-3}$

$= 0.122 + 20(0.1)(0.9)^{19} + \frac{\overset{10}{\cancel{20}} \times 19}{\cancel{2} \times 1} (0.1)^2 (0.9)^{18}$

$+ \frac{\overset{10}{\cancel{20}} \times 19 \times \overset{6}{\cancel{18}}}{\cancel{3} \times \cancel{2} \times 1} (0.1)^3 (0.9)^{17}$

$= 0.122 + 0.270 + 0.285 + 0.190$

$P(X \leq 3) = 0.867$