Probability:

Probability is a concept which we use to deal with uncertainty.

* we use probability in daily life to make decisions when you don't know for sure what the outcome will be.

For example,

1. Most probably it will rain today
2. I doubt that he will win the race.
3. chances are high that the prices of Petrol will go up.

Applications:

* modeling of text and web data
* speech recognition
* Robotics
* Network traffic and system Reliability modeling
* probabilistic analysis of algorithms and graphs
* machine learning and data mining
* Cryptography.

Terms related with Probability:

Experiment:

An experiment which, though repeated under essentially identical conditions does not give unique results but may result in any one of the several possible outcomes

Out come:

A result of an experiment is called an outcome.

Sample space:

A Sample Space is the collection of all possible out comes.

Trial:

Performing an experiment is known as trial

Event:

The outcomes of the experiment are known as Event.

Types of Events:

* Mutually Exclusive Events:

If the occurence of one event excludes the occurence of another event, such events are mutually exclusive events

* Exhaustive Events:

A set of events is called exhaustive, if all the events together consume the entire sample space.

* Independent event:

If the occurence of one event has no influence over the occurence of the other event.

Probability of an Event :

$$P(A) = \frac{\text{Favourable number of cases}}{[\text{Exhaustive number of cases. (or) Total number of cases}]}$$

1]. Find the probability of getting
i). 4
ii). an odd number in a die.

Soln. :

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(A) = \frac{1}{6} \; ; \; P(B) = \frac{3}{6} = \frac{1}{2}.$$

2] If a coin is tossed, then what is the probability of getting head?

$$S = \{H, T\}$$

$$P(\text{Head}) = \frac{1}{2}$$

3]. If you flip a balanced coin twice, what is the probability of getting atleast one head?

$$S = \{HH, HT, TH, TT\} \qquad 2^n. \quad 2^2 = 4$$

P(atleast one head)

$$= \frac{3}{4}$$

Axioms of probability:

1. For any event A, $P(A) \geq 0$
2. probability of the sample space S is $P(S) = 1$
3. If $A_1, A_2, \dots$ are disjoint events, then

$$P[A_1 \cup A_2 \cup \dots] = P[A_1] + P[A_2] + \dots \quad [\because A_i \cap A_j = \phi]$$

Results:

i). $P[\phi] = 0$ impossible event

ii). $P[\bar{A}] = 1 - P[A]$

iii) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

iv). $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

v). $P[A \cup B] = P(A) + P(B)$ A & B are mutually exclusive events

vi). $P(A \cap B) = P(A) \cdot P(B)$ A & B are independent

vii). $P[A \cup B] = P(A) + P(B) - P(A \cap B)$

viii). $P(A^c \cup B^c) = P(A \cap B)^c$; $P(A^c \cap B^c) = P(A \cup B)^c$

ix). $P(A+B) = P(A \cup B)$; $P(AB) = P(A \cap B)$

problems.

1. If $P(A) = 0.4$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$. find $P(\bar{A} \cap \bar{B})$ and $P(\bar{A} \cup \bar{B})$

Soln.:

$$P(\bar{A}) = 1 - P(A) = 1 - 0.4 = 0.6$$

$$P(\bar{B}) = 1 - P(B) = 1 - 0.7 = 0.3$$

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

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$$= 1 - [0.4 + 0.7 - 0.3] = 1 - 0.8 = 0.2$$

$P(\bar{A} \cap \bar{B}) = 0.2$

$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$$
$$= 1 - 0.3$$
$$P(\bar{A} \cup \bar{B}) = 0.7$$

2. If A and B are even with $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$. Find $P(A^c \cap B^c)$

Soln:

$$P(A^c \cap B^c) = P(A \cup B)^c$$
$$= 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - \left[\frac{3}{8} + \frac{1}{2} - \frac{1}{4}\right]$$
$$= 1 - \left(\frac{3+4-2}{8}\right) = 1 - \frac{5}{8} = \frac{8-5}{8}$$
$$= \frac{3}{8}$$

3. Event A & B are $P(A+B) = \frac{3}{4}$, $P(AB) = \frac{1}{4}$. $P(\bar{A}) = \frac{2}{3}$. Find $P(B)$

Soln.

Gvn. $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$

$P(\bar{A}) = \frac{2}{3}$ | $P(\bar{A}) = 1 - P(A) \Rightarrow P(A) = 1 - P(\bar{A})$

$P(A) = 1 - \frac{2}{3} = \frac{1}{3}$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(B) = P(A\cup B) - P(A) + P(A\cap B)$$

$$= \frac{3}{4} - \frac{1}{3} + \frac{1}{4} = \frac{9-4+3}{12}$$

$$P(B) = \frac{8}{12} = \frac{2}{3}$$

4]. A bag containing 6 red, 4 black and 7 blue, 10 white. Five balls are drawn at random. what is the probability that two of them are red and one is black, two is blue.

Soln. 6R . 4BLA 7B 10W = 27

$$\text{Prob} = \frac{6C_2 \times 4C_1 \times 7C_2}{27C_5}$$

$$= \frac{6\times5\times4\times7\times6\times5\times4\times3\times2\times1}{2\times1\times2\times1\times27\times26\times25\times24\times23} \quad (nC_1 = n)$$

$$= \frac{14}{897} = 0.015$$

5]. A bag containing 5 white balls, 6 green balls. Three balls are drawn with replacement. what is the chance that

i). All are same color

ii). They are alternatively different color.

i). All are same color

Prob = Either All are white or All are Green

$$= \frac{5C_3}{11C_3} + \frac{6C_3}{11C_3}$$

$$= \frac{5\times4\times3\times3\times2\times1}{3\times2\times1\times11\times10\times9} + \frac{6\times5\times4\times3\times2\times1}{3\times2\times1\times11\times10\times9}$$

$$= \frac{2}{33} + \frac{4}{33} = \frac{6}{33} = \frac{2}{11} = 0.18$$

ii). All are alternative different color

$$P(A) = P(WGW) + P(GWG)$$

$$= \frac{5C_2\ 6C_1}{11C_3} + \frac{6C_2\ 5C_1}{11C_3}$$

$$= \frac{5\times4\times6\times3\times2\times1}{2\times1\times11\times10\times9} + \frac{6\times5\times5\times3\times2\times1}{2\times1\times11\times10\times9}$$

$$= \frac{4}{11} + \frac{5}{11} = \frac{9}{11} = 0.8$$

Without Replacement:

i). $P(A) = \frac{5C_1}{11C_1} \times \frac{4C_1}{10C_1} \times \frac{3C_1}{9C_1} + \frac{6C_1}{11C_1} \times \frac{5C_1}{10C_1} \times \frac{4C_1}{9C_1}$

$$= \frac{5\times4\times3}{11\times10\times9} + \frac{6\times5\times4}{11\times10\times9}$$

$$= \frac{2}{33} + \frac{4}{33} = \frac{6}{33} = \frac{2}{11} = 0.18$$

ii). All are alternative different color

$$= P(WGW) + P(GWG)$$

$$= \frac{5C_1}{11C_1} \cdot \frac{6C_1}{10C_1} \cdot \frac{4C_1}{9C_1} + \frac{6C_1}{11C_1} \cdot \frac{5C_1}{10C_1} \cdot \frac{5C_1}{9C_1}$$

$$= \frac{\cancel{5} \times \cancel{6}^{3} \times 4}{11 \times \cancel{10}_{2} \times \cancel{9}_{3}} + \frac{\cancel{6}^{3} \times \cancel{5} \times 5}{11 \times \cancel{10}_{2} \times \cancel{9}_{3}}$$

$$= \frac{4}{33} + \frac{5}{33} = \frac{9}{33} = \frac{3}{11} = 0.27$$

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