Poisson Distribution:

A random variable x is said to follow Poisson distribution, if it assume only non negative values of its probability mass function is given by,

$$P[X=x] = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0 \text{ to } \infty \text{ and where } \lambda \text{ is a parameter, } \lambda = np.$$

Property:

* Binomial distribution is a limiting case of poisson distribution.
* $n \to \infty, \quad p \to 0$

MGF, mean and variance of Poisson Distribution:

MGF:

$$M_X(t) = \sum_{x=0}^{\infty} e^{tx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{e^{tx}\lambda^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^{t})^{x}}{x!}$$

$$= e^{-\lambda}\left[\frac{(\lambda e^{t})^{0}}{0!} + \frac{(\lambda e^{t})^{1}}{1!} + \frac{(\lambda e^{t})^{2}}{2!} + \cdots\right]$$

$$= e^{-\lambda}\left[1 + \frac{\lambda e^{t}}{1!} + \frac{(\lambda e^{t})^{2}}{2!} + \cdots\right]$$

$$= e^{-\lambda}\left[e^{e^{t}\lambda}\right] \qquad \left[\because e^{x} = 1 + \frac{x}{1!} + \frac{x^{2}}{2!} + \cdots\right]$$

$$= e^{-\lambda + \lambda e^{t}}$$

$$M_x(t) = e^{(e^t-1)\lambda}$$

mean:

$$E(x) = \left[\frac{d}{dt} M_x(t)\right]_{t=0}$$

$$= \left[\frac{d}{dt} e^{(e^t-1)\lambda}\right]_{t=0}$$

$$= \left[e^{(e^t-1)\lambda} \cdot \lambda e^t\right]_{t=0}$$

$$= \lambda e^{(1-1)\lambda}$$

$$= \lambda e^0$$

$$E(x) = \lambda$$

Variance:

$$\text{var}(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \left[\frac{d^2}{dt^2} M_x(t)\right]_{t=0}$$

$$= \left[\frac{d}{dt}\left[\lambda e^t \, e^{(e^t-1)\lambda}\right]\right]_{t=0}$$

$$= \lambda\left[e^t \, e^{(e^t-1)\lambda} \, \lambda e^t + e^{(e^t-1)\lambda} \, e^t\right]_{t=0}$$

$$= \lambda\left[e^{(1-1)\lambda} \, \lambda + e^{(1-1)\lambda}\right]$$

$$= \lambda[\lambda + 1]$$

$$E(x^2) = \lambda^2 + \lambda$$

$$\therefore \text{var}(x) = \lambda^2 + \lambda - \lambda^2 = \lambda$$

$$M_x(t) = e^{\lambda(e^t-1)}$$

$$\text{mean} = \lambda$$

$$\text{variance} = \lambda$$

1. If a poisson variate $P(X=2) = 9P(X=4) + 90P(X=6)$. Find mean and variance.

Soln.

P.D: $P(X=x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

Given, $P(X=2) = 9P(X=4) + 90P(X=6)$

$$\frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$$

$\div e^{\lambda}$ $$\frac{\lambda^2}{2} = \frac{9\lambda^4}{24} + \frac{90\lambda^6}{720}$$

$\div \lambda^2$ $$\frac{1}{2} = \frac{9\lambda^2}{24} + \frac{90\lambda^4}{720}$$

$$\frac{1}{2} = \frac{3\lambda^2}{8} + \frac{\lambda^4}{8}$$

$$\lambda^4 + 3\lambda^2 = 4$$

$\lambda^4 + 3\lambda^2 - 4 = 0$

$\lambda^3 + \lambda^2 + 4\lambda + 4 = 0$ and $\lambda = -1$

$$\begin{array}{c|ccccc} 1 & 1 & 0 & 3 & \cancel{} & -4 \\ & 0 & 1 & 1 & 4 & 4 \\ \hline & 1 & 1 & 4 & 4 & \boxed{0} \end{array}$$

$\therefore \lambda = -1, 2, -2$

$\Rightarrow \lambda = 2$

mean = variance = 2

---

2. If the MGF is $e^{4(e^t-1)}$, find $P(X = \mu + \sigma)$ where μ and σ^2 are mean and variance of poisson distribution.

Soln.

Given, $M_X(t) = e^{4(e^t-1)}$

$$e^{\lambda(e^t-1)} = e^{4(e^t-1)}$$

$$\Rightarrow \lambda = 4$$

CS Scanned with CamScanner

mean $= \lambda = 4 \Rightarrow$ mean $= \mu$

$\lambda = \mu$

$\therefore \mu = 4$

variance : $\lambda = 4$

$\Rightarrow$ variance $= \sigma^2$

$\lambda = \sigma^2$

$\therefore \sigma^2 = 4$

$\sigma = 2$

Now, $P(X = \mu + \sigma) = P(X = 4+2) = P(X = 6)$

PD : $P(X = x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$, $x = 0$ to ∞

$P(X=6) = \dfrac{e^{-4} 4^6}{6!}$

$= \dfrac{(0.018)4096}{720}$

$P(X=6) = 0.102$

3] The number of monthly breakdown of a computer is a random variable having Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month with only one breakdown.

Soln.

PD : $P[X=x] = \dfrac{e^{-\lambda} \lambda^x}{x!}$

Given, mean $\lambda = 1.8$

$\therefore P[X=x] = \dfrac{e^{-1.8} (1.8)^x}{x!}$

$$P[x=1] = \frac{e^{-1.8}(1.8)^1}{1}$$

$$= 0.165 \times 1.8$$

$$P[x=1] = 0.297$$

4]. If 3% of electric bulbs manufacture by a company are defective. Find the probability that in the sample of 100 bulbs exactly 5 bulbs are defective..

Soln.

PD : $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given, $n=100$, $p = 3\% = 0.03$

$\lambda = np$

$= 100\ (0.03)$

$\lambda = 3$

Now, $P(X=5) = \frac{e^{-3}(3)^5}{5!}$

$$= \frac{(0.05)(243)}{120}$$

$$= \frac{12.150}{120}$$

$$= 0.101$$

5]. A manufacturer of pins knows that 2% of the product are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective. What is the probability that box will fail to meet guarantee quality?

Soln.

Given $n = 100$, $p = 2\% = 0.02$

$\lambda = np$

$= 100 \times 0.02$

$\lambda = 2$

Now, $P(X \le 4) = 1 - P(X > 4)$

$P(X > 4) = 1 - P(X \le 4)$

$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)]$

$= 1 - e^{-2}\left[\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!}\right]$

$= 1 - e^{-2}\left[1 + 2 + \frac{4}{2} + \frac{8}{6} + \frac{16}{24}\right]$

$= 1 - 0.135\left[\frac{3+6+6+4+2}{3}\right]$

$= 1 - 0.135\left[\frac{21}{3}\right]$

$= 1 - 0.135(7)$

$= 1 - 0.945$

$= 0.055$

HW

J. A car hire from 2 cars which it hires out day by day. The number of demands for a car in each day is distributed with mean 1.5. Calculate the preposition of days in which
i). neither car is used
ii). Some demand is refused.