Exponential Distribution:

If x is a continuous random variable which follows an exponential distribution, then

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where λ is a parameter, $\lambda > 0$.

MGF, mean & variance:

MGF:

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$= \int_0^{\infty} e^{tx} \lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_0^{\infty} e^{-x(\lambda - t)}\, dx$$

$$= \lambda \left[\frac{e^{-(\lambda - t)x}}{-(\lambda - t)} \right]_0^{\infty}$$

$$= \frac{-\lambda}{\lambda - t} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{-\lambda}{\lambda - t} (0 - 1)$$

$$M_X(t) = \frac{\lambda}{\lambda - t}$$

Mean:

$$E(X) = \left[\frac{d}{dt} M_X(t) \right]_{t=0}$$

$$= \left[\frac{d}{dt} \left(\frac{\lambda}{\lambda - t} \right) \right]_{t=0}$$

$$= \left[\lambda\left(\frac{-1}{(\lambda-t)^2}(-1)\right)\right]_{t=0} \qquad \because \frac{d}{dt}\frac{1}{t} = -\frac{1}{t^2}$$

$$= \left[\frac{\lambda}{(\lambda-t)^2}\right]_{t=0}$$

$$= \frac{\lambda}{\lambda^2}$$

$$E(x) = \frac{1}{\lambda}$$

$$var(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \left[\frac{d^2}{dt^2} M_x(t)\right]_{t=0}$$

$$= \left[\frac{d}{dt}\left(\frac{\lambda}{(\lambda-t)^2}\right)\right]_{t=0} \qquad \frac{d}{dt}\left(\frac{1}{t^2}\right) = \frac{-2}{t^3}$$

$$= \lambda\left(\frac{-2(-1)}{(\lambda-t)^3}\right)_{t=0}$$

$$= \frac{2\lambda}{\lambda^3}$$

$$E(x^2) = \frac{2}{\lambda^2}$$

$$\therefore var(x) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

$$M_x(t) = \frac{\lambda}{\lambda - t}$$

$$mean = \frac{1}{\lambda}$$

$$Variance = \frac{1}{\lambda^2}$$

Memoryless Property:

If X is exponential distribution, then for any integers s and t, then prove that

$$P(X>s+t / X>s) = P(X>t)$$

Soln.

WKT

$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$

Now,

$$P(X>s+t / X>s) = \frac{P(X>s+t \cap X>s)}{P(X>s)}$$

$$= \frac{P(X>s+t)}{P(X>s)} \rightarrow (1)$$

$$P(X>s) = \int_s^\infty \lambda e^{-\lambda x}\,dx$$

$$= \lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_s^\infty$$

$$= -\left[e^{-\infty} - e^{-\lambda s}\right]$$

$$= -\left[0 - e^{-\lambda s}\right]$$

$$= e^{-\lambda s}$$

<u>Illy</u> $P(X>s+t) = e^{-\lambda(s+t)}$

$$= e^{-\lambda s - \lambda t}$$

$$= e^{-\lambda s} \cdot e^{-\lambda t}$$

$$(1) \Rightarrow P(X > s+t / X > s) = \frac{e^{-\lambda s} e^{-\lambda t}}{e^{-\lambda s}}$$

$$= e^{-\lambda t}$$

$$= P(X > t)$$

Hence proved

J. The time in hours required to repair a machine is exponential distribution with parameter $\lambda = \frac{1}{2}$.

i). what is the probability that repair time exceed 2 hrs.

ii). what is the probability that repair takes atleast 11 hrs. given that its duration exceed 8 hrs.

Soln.

Given $\lambda = \frac{1}{2}$

i). $P(X>2) = e^{-\lambda(2)}$ $\quad \because P(X>s) = e^{-\lambda s}$

$= e^{-\frac{1}{2}(2)}$

$= e^{-1}$

$= 0.368$

ii). $P(X > 8+3 / X > 8) = P(X > 3)$

$\because P(X > s+t / X > s) = P(X > t)$

$= e^{-\lambda(3)}$

$= e^{-3/2}$

$= 0.223$

2). The milage which the car owners get with certain kind of radial tyre is a random variable having an exponential distribution with mean 4000 km. Find the probability that one of these tyre will last

i). Atleast 2000 km

ii). Atmost 3000 km

Soln.

Given mean = 4000

$$\frac{1}{\lambda} = 4000$$

$$\lambda = \frac{1}{4000}$$

i). $P(X > 2000) = e^{-\lambda(2000)}$

$$= e^{-\frac{1}{4000}(2000)}$$

$$= e^{-1/2}$$

$$= 0.607$$

ii). $P(X \leq 3000) = 1 - P(X > 3000)$

$$= 1 - e^{-\lambda(3000)}$$

$$= 1 - e^{-\frac{1}{4000}(3000)}$$

$$= 1 - e^{-3/4}$$

$$= 0.528$$