Unit – III

Two Dimensional Random Variable

* Joint Distribution
* Marginal Distribution
* Conditional Distribution
* Covariance Distribution
* Correlation Distribution
* Regression Distribution
* Functions of Random variable.

Discrete

1. Joint probability mass function

i). $P(x_i, y_j) \geq 0$

ii). $\sum_{i=1}^{n} \sum_{j=1}^{n} P(x_i, y_j) = 1$

2. To find constant:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} P(x_i, y_j) = 1$$

3. Marginal distribution function of X:

$$P(x) = \sum_{j=1}^{n} P(x_i, y_j)$$

Marginal distribution function of Y:

$$P(y) = \sum_{i=1}^{n} P(x_i, y_j)$$

4. Cumulative distribution:

$$F(X, Y) = P(X \leq x, Y \leq y)$$

Continuous

1. Joint Probability Density Function

i). $f(x, y) \geq 0$

ii). $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dy\, dx = 1$

2. To find constant

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dy\, dx = 1$$

3. Marginal distribution function of X:

$$f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$$

Marginal distribution function of Y:

$$f(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$$

4. Cumulative distribution:

$$F(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(x, y)\, dy\, dx$$

Discrete

5]. To check X & Y are independent:

$$P(i, j) = P(X=i) \cdot P(Y=j)$$

6]. Conditional Distribution

$$P(X=x_i / Y=y_j) = \frac{P(X=x_i, Y=y_j)}{P(Y=y_j)}$$

$$P(Y=y_j / X=x_i) = \frac{P(X=x_i, Y=y_j)}{P(X=x_i)}$$

Continuous

5]. To check X & Y are independent

$$f(x, y) = f(x) \cdot f(y)$$

6]. Conditional Distribution

$$f(x/y) = \frac{f(x, y)}{f(y)}$$

$$f(y/x) = \frac{f(x, y)}{f(x)}$$

7]. Joint cumulative function is given then to find PDF

$$f(x, y) = \frac{\partial^2}{\partial x \partial y} F(x, y)$$

1. From the following table for bivariate distribution of (X, Y). Find

i). $P(X \le 1)$ ii). $P(Y \le 3)$ iii). $P(X \le 1, Y \le 3)$

iv). $P(X \le 1 / Y \le 3)$ v). $P(Y \le 3 / X \le 1)$

vi). Marginal distribution function of X & Y.

vii). Conditional distribution of X given $Y = 2$

viii). Estimate X & Y are independent

ix). $P(X + Y \le 4)$

Soln.

$X \backslash Y$	1	2	3	4	5	6	$P(X)$
0	0	0	$1/32$	$2/32$	$2/32$	$3/32$	$8/32$
1	$1/16$	$1/16$	$1/8$	$1/8$	$1/8$	$1/8$	$\frac{10}{16}$
2	$1/32$	$1/32$	$1/64$	$1/64$	0	$2/64$	$\frac{8}{64}$
$P(Y)$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$	1

i). $P(X \le 1)$

$$P(X \le 1) = P(X=0) + P(X=1)$$
$$= \frac{8}{32} + \frac{10}{16}$$
$$= \frac{28}{32} = \frac{7}{8}$$

ii). $P(Y \le 3)$

$$P(Y \le 3) = P(Y=1) + P(Y=2) + P(Y=3)$$
$$= \frac{3}{32} + \frac{3}{32} + \frac{11}{64}$$
$$= \frac{23}{64}$$

$X = 0, 1$

$Y = 1, 2, 3$

iii). $P(X \le 1,\ Y \le 3)$

$$P(X \le 1, Y \le 3) = P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)$$

$$= 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8}$$

$$= \frac{1+2+2+4}{32}$$

$$= \frac{9}{32}$$

iv). $P(X \le 1 / Y \le 3) = \dfrac{P(X \le 1, Y \le 3)}{P(Y \le 3)}$

$$= \frac{P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)}{P(Y=1) + P(Y=2) + P(Y=3)}$$

$$= \frac{0 + 0 + 1/32 + 1/16 + 1/16 + 1/8}{3/32 + 3/32 + 11/64}$$

$$= \frac{9/32}{23/64}$$

$$= \frac{18}{23}$$

v). $P(Y \le 3 / X \le 1) = \dfrac{P(X \le 1, Y \le 3)}{P(X \le 1)}$

$$= \frac{P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)}{P(X=0) + P(X=1)}$$

$$= \frac{0 + 0 + 1/32 + 1/16 + 1/16 + 1/8}{28/32}$$

$$= \frac{9/32}{28/32}$$

$$= \frac{9}{28}$$

vi). Marginal distribution function of X

X	0	1	2
$P(X)$	$8/32$	$10/16$	$8/64$

Marginal distribution function of Y:

Y	1	2	3	4	5	6
$P(Y)$	$3/32$	$3/32$	$11/64$	$13/64$	$6/32$	$16/64$

vii). Conditional distribution function of X on $Y=2$.

$P(X/Y=2)$:

$$P(X=0/Y=2) = \frac{P(X=0, Y=2)}{P(Y=2)} = 0$$

$$P(X=1/Y=2) = \frac{P(X=1, Y=2)}{P(Y=2)} = \frac{1/16}{3/32} = \frac{2}{3}$$

$$P(X=2/Y=2) = \frac{P(X=2, Y=2)}{P(Y=2)} = \frac{1/32}{3/32} = 1/3$$

viii). X & Y are independent.

$$\Rightarrow P(X=i, Y=j) = P(X=i) \cdot P(Y=j)$$

Consider $P(2,3)$

$$P(2,3) = P(X=2) \cdot P(Y=3)$$

$$\frac{1}{164} \neq \frac{8}{64} \cdot \frac{11}{64}$$

$\therefore$ X & Y are not independent.

ix). $P(X+Y \le 4)$

$$P(X+Y \le 4) = P(0,1) + P(0,2) + P(0,3) + P(0,4) + P(1,1) + P(1,2) + P(1,3) + P(2,1) + P(2,2)$$

$$= 0 + 0 + 1/32 + 2/32 + 1/16 + 1/16 + 1/8 + 1/32 + 1/32$$

$$= \frac{1+2+2+2+4+1+1}{32}$$

$$= \frac{13}{32}$$

2. If the joint PDF of (X,Y) is given by $P(x,y) = k(2x+3y)$, $x = 0,1,2$; $y = 1,2,3$. Find all the marginal probability distribution. Also find the prob. distribution of $(X+Y)$ and $P(X+Y > 3)$.

Soln.

Given $P(x,y) = k(2x+3y)$

Y \ X	0	1	2	P(Y)
1	3K	5K	7K	15K
2	6K	8K	10K	24K
3	9K	11K	13K	33K
P(X)	18K	24K	30K	72K

Total probability = 1

$$72K = 1$$

$$K = \frac{1}{72}$$

$Y \downarrow \backslash X \rightarrow$	0	1	2	$P(Y)$
1	$3/72$	$5/72$	$7/72$	$15/72$
2	$6/72$	$8/72$	$10/72$	$24/72$
3	$9/72$	$11/72$	$13/72$	$33/72$
$P(X) \rightarrow$	$\frac{18}{72}$	$\frac{24}{72}$	$\frac{30}{72}$	1

Marginal Probability function of X

$P(X=0) = 18/72$

$P(X=1) = 24/72$

$P(X=2) = \frac{30}{72}$

Marginal Probability function of Y

$P(Y=1) = \frac{15}{72}$

$P(Y=2) = \frac{24}{72}$

$P(Y=3) = \frac{33}{72}$

Probability Distribution of $X+Y$:

		Probability
$P(X+Y=1)$		$P(0,1) = 3/72$
$P(X+Y=2)$	$P(1,1)+P(0,2) = \frac{5}{72}+\frac{6}{72}$	$= \frac{11}{72}$
$P(X+Y=3)$	$P(2,1)+P(1,2)+P(0,3) = \frac{7}{72}+\frac{8}{72}+\frac{9}{72}$	$= \frac{24}{72}$
$P(X+Y=4)$	$P(1,3)+P(2,2) = \frac{11}{72}+\frac{10}{72}$	$= \frac{21}{72}$
$P(X+Y=5)$		$P(2,3) = \frac{13}{72}$

$$P(X+Y>3) = P(X+Y=4)+P(X+Y=5)$$
$$= \frac{21}{72} + \frac{13}{72}$$
$$= \frac{34}{72}$$

3. The two dimensional random variable (X, Y) has joint probability mass function $f(x, y) = \frac{x+2y}{27}$, $x = 0, 1, 2$; $y = 0, 1, 2$. Find the conditional distribution of Y for $X = x$.
Also find conditional distribution of Y given $X = 1$.

Soln.

Given $f(x, y) = \frac{x+2y}{27}$

$X \backslash Y$	0	1	2	$P(X)$
0	0	2/27	4/27	6/27
1	1/27	3/27	5/27	9/27
2	2/27	4/27	6/27	12/27
$P(Y)$	3/27	9/27	15/27	1

i). $P(Y/X=x)$:

When $X=0$,
$$P(Y=0/X=0) = \frac{P(X=0, Y=0)}{P(X=0)} = \frac{0}{6/27} = 0$$

$$P(Y=1/X=0) = \frac{P(X=0, Y=1)}{P(X=0)} = \frac{2/27}{6/27} = \frac{2}{6}$$

$$P(Y=2/X=0) = \frac{P(X=0, Y=2)}{P(X=0)} = \frac{4/27}{6/27} = \frac{4}{6}$$

when $x=1$,

$$P(y=0/x=1) = \frac{P(x=1, y=0)}{P(x=1)} = \frac{1/27}{9/27} = \frac{1}{9}$$

$$P(y=1/x=1) = \frac{P(x=1, y=1)}{P(x=1)} = \frac{3/27}{9/27} = \frac{3}{9}$$

$$P(y=2/x=1) = \frac{P(x=1, y=2)}{P(x=1)} = \frac{5/27}{9/27} = \frac{5}{9}$$

when $x=2$,

$$P(y=0/x=2) = \frac{P(x=2, y=0)}{P(x=2)} = \frac{2/27}{12/27} = \frac{2}{12}$$

$$P(y=1/x=2) = \frac{P(x=2, y=1)}{P(x=2)} = \frac{4/27}{12/27} = \frac{4}{12}$$

$$P(y=2/x=2) = \frac{P(x=2, y=2)}{P(x=2)} = \frac{6/27}{12/27} = \frac{6}{12}$$

ii). $P(y/x=1)$

$$P(y=0/x=1) = \frac{1}{9}$$

$$P(y=1/x=1) = \frac{3}{9}$$

$$P(y=2/x=1) = \frac{5}{9}$$