



DEPARTMENT OF MATHEMATICS

Method of variation of parameters :

The Second order linear differential equation is,

$$\frac{d^2 y}{dx^2} + P \frac{dy}{dx} + Q y = R$$

Step 1 : Find Complimentary function. From this calculate y_1 and y_2 (Coefficient of constant).

Step 2 : Find Wronskian.

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

$$P.I = P y_1 + Q y_2$$

$$\text{where } P = - \int \frac{R}{W} y_2 dx$$

$$Q = \int \frac{R}{W} y_1 dx$$

① Find Wronskian for the following :

(i) $\frac{d^2 y}{dx^2} + 4y = \operatorname{cosec} x$

(ii) $y'' + y = \sec^2 x$

(iii) $(D^2 + 16)y = \sec 4x$

Soln :

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$$(1) \frac{d^2 y}{dx^2} + 4y = \operatorname{cosec} x$$

The A.E is

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i = 0 \pm 2i$$

$$\alpha = 0, \beta = 2$$

$$C.F = e^{\alpha x} (A \cos \beta x + B \sin \beta x)$$

$$= e^{0x} (A \cos 2x + B \sin 2x)$$

$$C.F = A \cos 2x + B \sin 2x$$

$$y_1 = \cos 2x, \quad y_2 = \sin 2x$$

$$y_1' = -\sin 2x (2) \quad ; \quad y_2' = \cos 2x \cdot 2$$
$$= -2 \sin 2x \quad \quad \quad = 2 \cos 2x$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \sin 2x & 2 \cos 2x \end{vmatrix}$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$= 2 (\cos^2 2x + \sin^2 2x)$$

$$\boxed{W = 2}$$