Method of variation of parameters :

The Second order linear differential equation is,

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Step 1 : Find complimentary function. From this calculate y_1 and y_2 (Coefficient of constant)

Step 2 : Find Wronskian.

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

$$P.I = Py_1 + Qy_2$$

where $P = -\int \frac{R}{W} y_2\, dx$

$$Q = \int \frac{R}{W} y_1\, dx$$

(1) Find Wronskian for the following :

(i) $\frac{d^2y}{dx^2} + 4y = \text{Cosec } x$

(ii) $y'' + y = \text{Sec}^2 x$

(iii) $(D^2 + 16)y = \text{Sec } 4x$

Soln :

(1) $\frac{d^2y}{dx^2} + 4y = \operatorname{cosec} x$

The A.E is

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm 2i = 0 \pm 2i$

$\alpha = 0, \beta = 2$

$C.F = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$= e^{0x}(A\cos 2x + B\sin 2x)$

$C.F = A\cos 2x + B\sin 2x$

$y_1 = \cos 2x, \quad y_2 = \sin 2x$

$y_1' = -\sin 2x\,(2)$; $y_2' = \cos 2x \cdot 2$

$= -2\sin 2x$ $= 2\cos 2x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$= 2\cos^2 2x + 2\sin^2 2x$

$= 2(\cos^2 2x + \sin^2 2x)$

$\boxed{W = 2}$