



## DEPARTMENT OF MATHEMATICS

### UNIT - III

### COMPLEX DIFFERENTIATION

#### INTRODUCTION:

If  $x$  and  $y$  are real numbers then  $z = x + iy$  is called a complex number where  $x$  is called real part of  $z$ ,  $y$  is called the imaginary part of  $z$  and the value of  $i$  is  $\sqrt{-1}$ . The complex number  $x - iy$  is called as the complex conjugate of  $z$  & it is denoted by  $\bar{z}$ .  
i.e.,  $\bar{z} = x - iy$ .

#### NOTE:

1.  $|z| = \sqrt{x^2 + y^2}$
2.  $|z^2| = z\bar{z}$
3.  $z\bar{z} = x^2 + y^2 = r^2$
4.  $|\bar{z}| = |z|$
5. Real part of  $z = \frac{z + \bar{z}}{2}$
6. Imaginary part of  $z = \frac{z - \bar{z}}{2i}$
7.  $z = re^{i\theta}$  is called polar form of  $z$ .
8. Amplitude of  $z = \theta = \tan^{-1}(y/x)$

#### FUNCTIONS OF COMPLEX VARIABLE:

$w = f(z) = u(x, y) + iv(x, y)$  where  $u(x, y)$  and  $v(x, y)$  are real variables.



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### Single Valued function:

If for each value of  $z$  in  $R$  there will be only one value of  $w$ , then  $w$  is called a single valued function of  $z$ .

Ex:  $w = z^2$ ,  $w = 1/z$

$z: 1 \quad 2 \quad -2 \quad 3$

$w: 1 \quad 4 \quad 4 \quad 9$

$z: 1 \quad 2 \quad -2 \quad 3$

$w: 1 \quad 1/2 \quad -1/2 \quad 1/3$

### Multiple-valued function:

If there is more than one value of  $w$  corresponding to a given value of  $z$ , then  $w$  is called a multiple-valued function.

Ex:  $w = z^{1/2}$

$z: 4 \quad 9 \quad 1$

$w: -2, 2 \quad -3, 3 \quad 1, -1$

### Analytic function:

A function  $f(z)$  is said to be analytic at a point  $z = a$  in a region  $R$  if

- (i)  $f(z)$  is differentiable at  $z = a$ .
- (ii)  $f(z)$  is differentiable at all points for some neighbourhood of  $z = a$ .

(or)

A function is said to be analytic at a point if its derivative exists not only at that point but also in some neighbourhood of that point.