UNIT - III

COMPLEX DIFFERENTIATION

INTRODUCTION:

If x and y are real numbers then $z = x+iy$ is called a complex number where x is called real part of z, y is called the imaginary part of z and the value of i is $\sqrt{-1}$. The complex number $x-iy$ is called as the complex conjugate of z & it is denoted by $\bar{z}$.

i.e., $\bar{z} = x - iy$.

NOTE:

1. $|z| = \sqrt{x^2+y^2}$
2. $|z^2| = z\bar{z}$
3. $z\bar{z} = x^2+y^2 = r^2$
4. $|\bar{z}| = |z|$
5. Real part of $z = \dfrac{z+\bar{z}}{2}$
6. Imaginary part of $z = \dfrac{z-\bar{z}}{2i}$
7. $z = re^{i\theta}$ is called polar form of z.
8. Amplitude of $z = \theta = \tan^{-1}(y/x)$

FUNCTIONS OF COMPLEX VARIABLE:

$w = f(z) = u(x,y) + iv(x,y)$ where $u(x,y)$ and $v(x,y)$ are real variables.

Single Valued function:

If for each value of z in R there will be only one value of w, then w is called a single valued function of z.

Ex: $w = z^2$, $w = 1/z$.

z:	1	2	-2	3
w:	1	4	4	9

$w = 1/z$

z:	1	2	-2	3
w:	1	$1/2$	$-1/2$	$1/3$

Multiple-valued function:

If there is more than one value of w corresponding to a given value of z, then w is called a multiple-valued function.

Ex: $w = z^{1/2}$

z:	4	9	1
w:	$-2, 2$	$-3, 3$	$1, -1$

Analytic function:

A function $f(z)$ is said to be analytic at a point $z = a$ in a region R if

(i) $f(z)$ is differentiable at $z = a$.

(ii) $f(z)$ is differentiable at all points for some neighbourhood of $z = a$.

(or)

A function is said to be analytic at a point if its derivative exists not only at that point but also in some neighbourhood of that point.