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Necessary Condition (Cartesian Coordinates):
(or) Cauchy - Riemann equations:
If the function
$$f(z) = u(x,y) + iv(x,y)$$

is analytic in a Stegion R of the Z plane, then
(i) $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ exists
(ii) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
at every point in that region.
Sufficient Conditions:
If the function $f(z) = u(x,y) + iv(x,y)$ is
analytic in a region R of the Z-plane if
(i) u_x , u_y , $v_x \ge v_y$ are exists and all are continuous.
(ii) $u_x = v_y$ and $u_y = -v_x$.
Necessary Condition (polar Coordinates):
If the function $w = f(z) = u(r,0) + iv(r,0)$
is analytic in a sugion R of the Z-plane then
(i) if $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial x} = xists$.
If the function $w = f(z) = u(r,0) + iv(r,0)$
is analytic in a sugion R of the Z-plane, then
(i) $\frac{\partial u}{\partial x} = \frac{1}{x} \frac{\partial v}{\partial 0}$ and $\frac{\partial v}{\partial 0} = xists$.
If the function $w = f(z) = u(r,0) + iv(r,0)$
is analytic in a sugion R of the Z-plane, then
(i) $\frac{\partial u}{\partial r}$, $\frac{\partial u}{\partial r}$, $\frac{\partial v}{\partial r}$ and $\frac{\partial v}{\partial \theta} = xists$.
(ii) $\frac{\partial u}{\partial r} = \frac{1}{x} \frac{\partial v}{\partial 0}$ and $\frac{\partial v}{\partial \theta} = xists$.
(ii) $\frac{\partial u}{\partial r} + \frac{\partial v}{\partial 0}$ and $\frac{\partial v}{\partial \theta} = xists$.
(ii) $\frac{\partial u}{\partial r} + \frac{\partial u}{\partial 0}$, $\frac{\partial v}{\partial r}$ and $\frac{\partial v}{\partial \theta} = \frac{1}{y} \frac{\partial u}{\partial \theta}$.
(ii) $\frac{\partial u}{\partial r} + \frac{\partial u}{\partial r}$ and $\frac{\partial v}{\partial \theta} = xists$ & all are continuous
(ii) $\frac{\partial u}{\partial r} = \frac{1}{x} \frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial \theta} = xists$ & all are continuous





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PROBLEMS:
(1) Prove that
$$w = z^{2}$$
 is analytic.
Soln:
 $w = x^{2}$
 $= (z + iy)^{2}$
 $= x^{2} - y^{2} + 2ixy$
 $u + iv = (x^{2} - y^{2}) + i(2xy)$
 $u = x^{2} - y^{2}$; $v = 2xy$
 $u_{z} = 2x$
 $u_{z} = v_{y}$ & $u_{y} = -v_{x}$
 $u_{x} = v_{y}$ & $u_{y} = -v_{x}$
 $u_{x} = v_{y}$ & $u_{y} = -v_{x}$
 $\exists r \cdot \delta atisfies CR equations:
 $\Rightarrow w = z^{3}$ is analytic.
(2) Determine whether the function $w = 2xy + i(x^{2} - y^{2})$
 $u = 2xy + i(x^{2} - y^{2})$
 $u = 2xy + i(x^{2} - y^{2})$
 $u_{x} = 2y$, $v_{x} = 2x$
 $u_{y} = 2x$, $v = x^{2} - y^{2}$
 $u_{x} = 2y$, $v = x^{2} - y^{2}$
 $u_{x} = y$
 $u_{x} \neq v_{y}$
 $w = 2xy + i(x^{2} - y^{2})$ is not analytic$





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$$u_{\chi} = 2\chi , \quad V_{\chi} = 0$$

$$u_{y} = \lambda y , \quad V_{y} = 0$$

$$\Rightarrow \quad u_{\chi} \neq V_{y} \& \quad u_{y} \neq -V_{\chi}$$
The does n't statisfies C.R. comes.

$$\therefore \quad f(z) = (z)^{2} \text{ is nowhere analytic}$$
(5) If $u + iv$ is analytic then $v - iu$ is also analytic.

$$i e, \quad U_{\chi} = v_{y} \& u_{y} = -v_{\chi}$$

$$i e, \quad U_{\chi} = v_{y} \& u_{y} = -v_{\chi}$$

$$i e, \quad U_{\chi} = v_{y} \& u_{y} = -\lambda$$
To prove: $v - iu$ is also analytic.

$$i e., \quad We have to prove, \quad \frac{\partial v}{\partial x} = \frac{\partial (-u)}{\partial y} \& \frac{\partial v}{\partial y} = \frac{\partial (-u)}{\partial x}$$
We know that,

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} \quad and \quad \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$
We know that,

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} \quad and \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \& \frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \quad and \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$
Hence viu is also analytic.
(6) The $w = e^{z}$, find $\frac{dw}{dz} = u_{y}^{2} (\cos y + i \sin y)$

$$= e^{x} \cos y , \quad v_{x} = e^{x} \sin y$$

$$u_{\chi} = e^{x} \cos y , \quad v_{\chi} = e^{x} \sin y$$





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$$\begin{aligned} & \left[\text{Result} : \text{If } \omega = f(z) = u + iv \text{ then } = -1 \\ & \frac{dw}{dz} = f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \\ & \text{-finding } \frac{dw}{dz} \text{ in terms of partial destivatives } w.r.tx) \\ & \Rightarrow \frac{dw}{dz} = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \\ & = e^{\pi} \cos y + i e^{\pi} \sin y \\ & = e^{\pi} (\cos y + i \sin y) \\ & = e^{\pi} e^{iy} \\ & = e^{\pi} e^{iy} \\ & = e^{\pi} e^{iy} \end{aligned}$$