SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT-III
COMPLEX DIFFERENTIATION

INTRODUCTION:

If x and y are real numbers then $Z = x+iy$ is called a complex number where x is called real part of z, y is called the imaginary part of z and the value of i is $\sqrt{-1}$. The complex number $x-iy$ is called as the complex conjugate of z & it is denoted by $\bar{z}$.

i.e., $\bar{z} = x - iy$.

NOTE:

1. $|z| = \sqrt{x^2+y^2}$
2. $|z^2| = z\bar{z}$
3. $z\bar{z} = x^2+y^2 = r^2$
4. $|\bar{z}| = |z|$
5. Real part of $z = \dfrac{z+\bar{z}}{2}$
6. Imaginary part of $z = \dfrac{z-\bar{z}}{2i}$
7. $z = re^{i\theta}$ is called polar form of z.
8. Amplitude of $z = \theta = \tan^{-1}(y/x)$

FUNCTIONS OF COMPLEX VARIABLE:

$w = f(z) = u(x,y) + iv(x,y)$ where $u(x,y)$ and $v(x,y)$ are real variables.

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DEPATMENT OF MATHEMATICS

Single Valued function:

If for each value of z in R there will be only one value of ω, then ω is called a single valued function of z.

Ex: $\omega = z^2$, $\omega = 1/z$.

z :	1	2	-2	3
ω :	1	4	4	9

$\omega = 1/z$

z :	1	2	-2	3
ω :	1	$1/2$	$-1/2$	$1/3$

Multiple - valued function:

If there is more than one value of ω corresponding to a given value of z, then ω is called a multiple - valued function.

Ex: $\omega = z^{1/2}$

z :	4	9	1
ω :	-2, 2	-3, 3	1, -1

Analytic function:

A function $f(z)$ is said to be analytic at a point $z = a$ in a Region R if

(i) $f(z)$ is differentiable at $z = a$.

(ii) $f(z)$ is differentiable at all points for some neighbourhood of $z = a$.

(or)

A function is said to be analytic at a point if its derivative exists not only at that point but also in some neighbourhood of that point.