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#### **DEPARTMENT OF MATHEMATICS**

Harmonic functions: An expression of the form 200. is called the Laplace equation in two dimension Any function having continuous second order postial derivatives which satisfies the Laplace equation is called harmonic function. Any two harmonic functions u and v such that f(z) = utiv is analytic are called Conjugate harmonic functions.





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# Note:

Both real and imaginary parts of an analytic function are harmonic. But the converse need not be true.

O Give an example such that u and v are harmonic but utiv is not analytic.

Let 
$$w = \overline{x}$$
 $u + iv = x - iy$ 

$$\Rightarrow u = x$$
 $\frac{\partial u}{\partial x} = 1$ 
 $\frac{\partial u}{\partial y} = 0$ 
 $\frac{\partial v}{\partial x} = 0$ 
 $\frac{\partial v}{\partial x} = 0$ 
 $\frac{\partial v}{\partial y} = 0$ 
 $\frac{\partial^2 u}{\partial x^2} = 0$ 
 $\frac{\partial^2 u}{\partial y^2} = 0$ 
 $\frac{\partial^2 u}{\partial y^2} = 0$ 
 $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ 

and  $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ .

$$\Rightarrow u \text{ and } v \text{ are harmonic.}$$

=> u and v are harmonion sinoured a 21 V

But  $u_{\chi} \neq V_{y}$  and  $u_{y} = -V_{\chi}$  f(z) = u + iv is not analytic.

2) Prove that  $u = e^{x} \cos y$  is a harmonic function. Soln:

Soln:

$$\frac{\partial u}{\partial x} = e^{x} \cos y ; \quad \frac{\partial u}{\partial y} = -e^{x} \sin y$$

$$\frac{\partial^{2} u}{\partial x^{2}} = e^{x} \cos y ; \quad \frac{\partial^{2} u}{\partial y^{2}} = -e^{x} \cos y$$

$$\Rightarrow \frac{\partial^{2} u}{\partial x^{2}} + \frac{\partial^{2} u}{\partial y^{2}} = 0$$

$$\vdots \quad u \quad \text{is a harmonic function}.$$

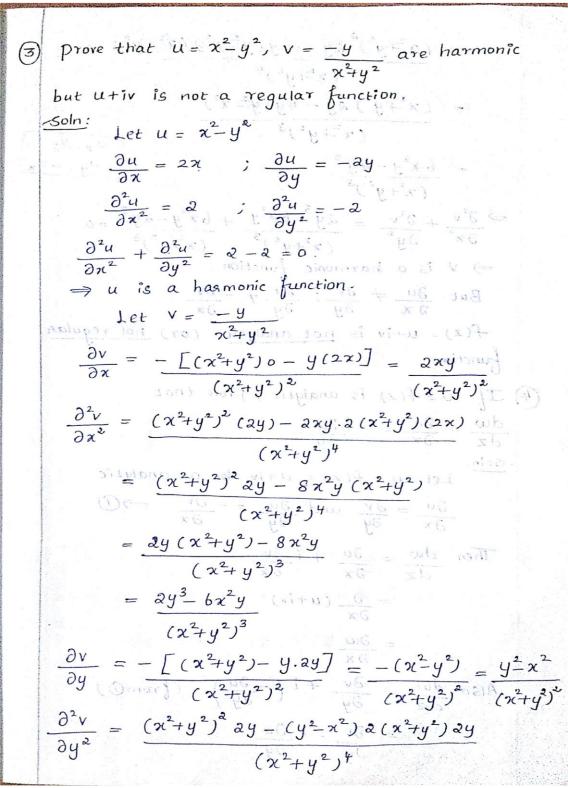




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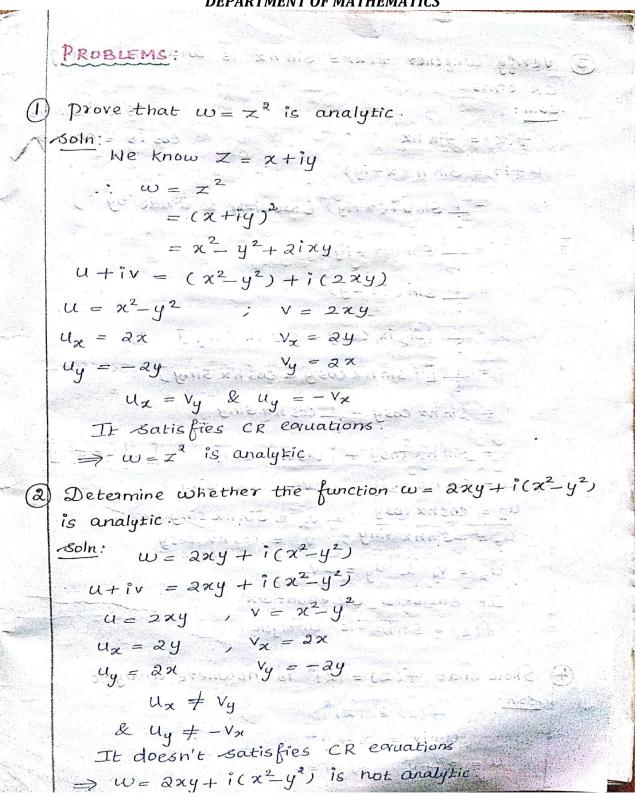




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