

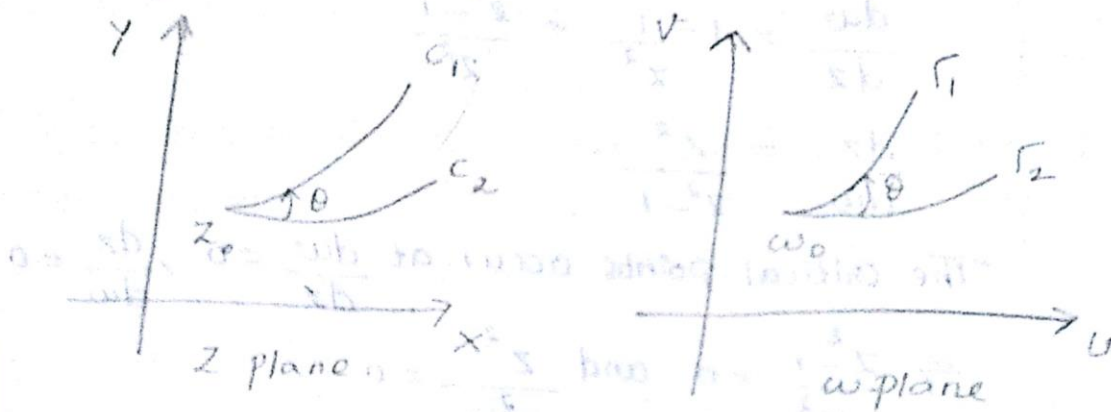


DEPARTMENT OF MATHEMATICS

Conformal mapping:

Defn:

A mapping $w = f(z)$ is said to be conformal at $z = z_0$ if it preserves the angle between any two curves through z_0 in z plane both in magnitude and direction.



Isogonal mapping:

A mapping $w = f(z)$ is said to be isogonal at $z = z_0$ if it preserves the angle between any two curves through z_0 in z plane only in magnitude



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but not in direction.

Remarks:

1. If $f(z)$ is analytic and $f'(z) \neq 0$ at each point then the mapping $w = f(z)$ is conformal.
2. The points at which $w = f(z)$ is not conformal i.e., $f'(z) = 0$ are called critical points.
3. The critical points of $w = f(z)$ will occur at $\frac{dz}{dw} = 0$ and $\frac{dw}{dz} = 0$.

① Find the critical points of the transformation $w = z + i/z$.

Soln:

$$\frac{dw}{dz} = 1 - \frac{1}{z^2} = \frac{z^2 - 1}{z^2}$$

$$\frac{dz}{dw} = \frac{z^2}{z^2 - 1}$$

The critical points occur at $\frac{dw}{dz} = 0$, $\frac{dz}{dw} = 0$.

$$\Rightarrow \frac{z^2 - 1}{z^2} = 0 \text{ and } \frac{z^2}{z^2 - 1} = 0$$

$$\Rightarrow z^2 - 1 = 0 \text{ and } z^2 = 0$$

$$\Rightarrow z = \pm 1 \text{ and } z = 0$$

$\therefore z = 0, 1, -1$ are the critical points.



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(2) Find the critical points of $w^2 = (z-\alpha)(z-\beta)$.

Soln:

$$w^2 = (z-\alpha)(z-\beta)$$

$$2w \frac{dw}{dz} = (z-\alpha) + (z-\beta)$$

$$\frac{dw}{dz} = \frac{(z-\alpha) + (z-\beta)}{2w}$$

$$\frac{dz}{dw} = \frac{2w}{(z-\alpha) + (z-\beta)}$$

$$\frac{dw}{dz} = \frac{(z-\alpha) + (z-\beta)}{2w} \quad \& \quad \frac{dz}{dw} = \frac{2w}{(z-\alpha) + (z-\beta)}$$

\therefore The critical points occur at $\frac{dw}{dz} = 0$ and $\frac{dz}{dw} = 0$.

$$\Rightarrow \frac{z-\alpha + z-\beta}{2w} = 0 \quad \text{and} \quad \frac{2w}{(z-\alpha) + (z-\beta)} = 0$$

$$\Rightarrow 2z = \alpha + \beta$$

$$\Rightarrow z = \frac{\alpha + \beta}{2}$$

$$w = 0$$

$$\Rightarrow w^2 = 0$$

$$\Rightarrow (z-\alpha)(z-\beta) = 0$$

$$\Rightarrow z = \alpha, \beta$$

$\therefore z = \alpha, \beta, \frac{\alpha + \beta}{2}$ are the critical points.

(3) Find the points such that $w = f(z) = \sin z$ is not conformal.

Soln: Let $w = \sin z$

$$\frac{dw}{dz} = \cos z; \quad \frac{dz}{dw} = \frac{1}{\cos z}$$

The critical points occur at $\frac{dw}{dz} = 0$ and $\frac{dz}{dw} = 0$.

$$\Rightarrow \cos z = 0 \quad \text{and} \quad 1/\cos z = 0$$

$$\Rightarrow z = \cos^{-1}(0)$$

$$1 = 0, \text{ impossible}$$

$$z = \frac{(2n-1)\pi}{2}, \quad n = 0, 1, 2, \dots$$