



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Method of variation of parameter

Method of variation of Parameter:

An equ which is in the form of $\frac{d^2y}{dx^2} + P \cdot \frac{dy}{dx} + Qy = R(x)$, then we can solve by using method of variation of Parameter, where P, Q, R are fn.

Methods to find solutions:

Step 1: Find the C.F of the given differential equation from this find y_1 and y_2

Step 2: Find $w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$ where w is wronskian.

Step 3: To find particular integral.

$$P.I = Ay_1 + By_2$$

$$\text{where } A = - \int \frac{R(x)}{w} y_2 dx$$

$$B = \int \frac{R(x)}{w} y_1 dx$$

Step 4: The general solution is $y = C.F + P.I$

Notes:

- 1) $\int \cot x \cdot dx = \log(\sin x)$
- 2) $\int \tan x \cdot dx = \log(\sec x)$
- 3) $\int \operatorname{cosec} x \cdot dx = -\log(\operatorname{cosec} x + \cot x)$
- 4) $\int \sec x \cdot dx = \log(\sec x + \tan x)$

Example: 1

Solve $\frac{d^2y}{dx^2} + 4y = 4(\tan 2x)$

Soln: $(D^2 + 4)y = 4(\tan 2x)$

$$\text{The A.E is } m^2 + 4 = 0$$

$$m^2 = -4$$

$$m^2 = i^2 \cdot 2^2$$

$$m = \pm 2i$$

$$\alpha = 0, \beta = 2$$

$$C.F = e^{0x} [A \cos 2x + B \sin 2x] = A \cos 2x + B \sin 2x$$

$$y_1 = \cos 2x$$

$$y_1' = -2 \sin 2x$$

$$y_2 = \sin 2x$$

$$y_2' = 2 \cos 2x$$



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$$k = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$= \begin{vmatrix} 2\cos^2 2x + 2\sin^2 2x \\ 2(\cos^2 2x + \sin^2 2x) \end{vmatrix} \quad \therefore [k] = 2$$

P.I = $Ay_1 + By_2$

$$A = -\int \frac{R(x)}{k} y_2 dx = -\int \frac{4 \tan 2x}{2} \sin 2x dx = -2 \int \tan 2x \cdot \sin 2x dx$$

$$= -2 \int \frac{\sin^2 2x}{\cos 2x} dx = -2 \int \frac{1 - \cos^2 2x}{\cos 2x} dx$$

$$= -2 \left[\int \frac{1}{\cos 2x} dx - \int \frac{\cos^2 2x}{\cos 2x} dx \right]$$

$$= -2 \left[\int \sec 2x dx - \int \cos 2x dx \right]$$

$$= -2 \left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2} \right]$$

$$A = \sin 2x - \log(\sec 2x + \tan 2x)$$

$$B = -\int \frac{R(x)}{k} y_1 dx = -\int \frac{4 \tan 2x}{2} \cos 2x dx = -2 \int \tan 2x \cdot \cos 2x dx$$

$$= -2 \int \frac{\sin 2x}{\cos 2x} \cdot \cos 2x dx = -2 \int \sin 2x dx = -\frac{\cos 2x}{2}$$

$$B = -\frac{\cos 2x}{2}$$

P.I = $Ay_1 + By_2$

$$= (\sin 2x - \log(\sec 2x + \tan 2x)) \cos 2x - \frac{\cos 2x}{2} (\sin 2x)$$

$$y = C.F + P.I$$

$$y = (\cos 2x + \sin 2x + \sin 2x - \log(\sec 2x + \tan 2x) \cos 2x - \frac{\cos 2x}{2} (\sin 2x))$$

Example : 2

P. $(D^2 + 1)y = x \cos x$

Solu: Given $(D^2 + 1)y = x \cos x$

The A.E is $m^2 + 1 = 0$

$$m^2 = -1$$

$$m = \pm i$$



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$C.F = e^{0x} [A \cos x + B \sin x]$
 $P.I = Ay_1 + By_2$
 $A P.I = - \int \frac{R(x)}{W} y_2 dx$
 $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \quad y_1 = \cos x \quad y_2 = \sin x$
 $\quad \quad \quad \quad \quad \quad \quad y_1' = -\sin x \quad y_2' = \cos x$
 $W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$
 $A P.I = - \int \frac{\sec x}{1} \sin x dx$
 $= - \int \frac{\sin x}{\cos x} dx$
 $= - \int \tan x dx = -\log |\sec x|$
 $P.I = -\log |\sec x|$
 $Q(x) = \int x^n dx = \frac{x^{n+1}}{n+1} = \frac{x^1}{1}$
 $\nabla Q = \int \frac{\sec x}{\cos x} \cos x dx$
 $= \int \frac{1}{\cos x} \cdot \cos x dx$
 $Q = \int dx = x$
 $P.I = P y_1 + Q y_2$
 $= -\log |\sec x| \cos x + x \sin x$
 $y = C.F + P.I$
 $= A \cos x + B \sin x - \log |\sec x| \cos x + x \sin x$

(x) Example : 3
 $(D^2 + 1)y = \cos \sec x$
Soln: Given, $(D^2 + 1)y = \cos \sec x$
 The A.E is $m^2 + 1 = 0$
 $m = \pm i$
 $\alpha = 0, \beta = 1$
 $\therefore C.F = e^{0x} [A \cos x + B \sin x]$
 $P.I = Ay_1 + By_2$
 $A = - \int \frac{R(x)}{W} y_2 dx$
 $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$
 $= \cos^2 x + \sin^2 x = 1$



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$$A = - \int \frac{\cos x \sec x}{1} \cos x dx$$
$$= - \int \frac{1}{\sin x} \cos x dx$$

$$= - \int \cot x dx$$

$$= - \log(\sin x)$$

$$B = \int \frac{R(x)}{w} y_1 dx$$

$$= \int \frac{\cos x \sec x}{1} \sin x dx$$

$$= \int \frac{1}{\sin x} \sin x dx$$

$$= \int dx$$

$$B = x$$

$$P.I = Ay_1 + By_2$$

$$= -\log(\sin x) \cos x + x \sin x$$

$$y = C.F + P.I$$

$$= e^{0x} [A \cos x + B \sin x] - \log(\sin x)$$