



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

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### Method of variation of Parameter:

An equ which is in the form of  
 $\frac{d^2y}{dx^2} + P \cdot \frac{dy}{dx} + Qy = R(x)$ , then we can solve  
 by using method of variation of Parameter,  
 where P, Q, R are fn.

### Methods to find solutions:

Step 1: Find the C.F of the given differential  
 equation from this find  $y_1$  and  $y_2$

Step 2: Find  $w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$  where  
 w is covariantian.

Step 3: To find particular integral.

$$P_1 = Ay_1 + By_2$$

$$\text{where } A = \int \frac{R(x)}{w} y_2 dx, B = \int \frac{R(x)}{w} y_1 dx$$

$$B = \int \frac{R(x)}{w} \cdot y_1 dx$$

Step 4: The general solution is  $y = C \cdot F + P \cdot I$

### Notes:

$$1) \int \cot x \cdot dx = \log(\sin x)$$

$$2) \int \tan x \cdot dx = \log(\sec x)$$

$$3) \int \operatorname{cosec} x \cdot dx = -\log(\operatorname{cosec} x + \cot x)$$

$$4) \int \sec x \cdot dx = \log(\sec x + \tan x)$$

### Example: 1

$$\text{Solve } \frac{d^2y}{dx^2} + 4y = 4(\tan 2x)$$

$$\text{Soln: } (D^2 + 4)y = 4(\tan 2x)$$

$$\text{The A.E is } m^2 + 4 = 0$$

$$m^2 = -4$$

$$m^2 = i^2$$

$$m = \pm 2i$$

$$\alpha = 0, \beta = 2$$

$$C.F = e^{0x} [A \cos 2x + B \sin 2x] = A \cos 2x + B \sin 2x$$

$$y_1 = \cos 2x$$

$$y_1' = -2 \sin 2x$$

$$y_2 = \sin 2x$$

$$y_2' = 2 \cos 2x$$



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## UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

### Method of variation of parameter

$$\begin{aligned}
 W &= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} \\
 &= \begin{vmatrix} 2\cos^2 2x + 2\sin^2 2x \\ 2(\cos^2 2x + \sin^2 2x) \end{vmatrix} \quad \therefore (W = 2)
 \end{aligned}$$

P.D =  $Ay_1 + By_2$

$$\begin{aligned}
 A &= -\int \frac{R(x)}{W} y_1 dx = \frac{1}{2} \left[ \frac{1}{\cos 2x} + \frac{1}{\sin 2x} \right] \\
 &\quad - \int \frac{4\tan 2x \cdot \sin 2x}{\cos 2x} dx = -2 \int \tan 2x \cdot \sin 2x dx \\
 &= -2 \int \frac{\sin^2 2x}{\cos 2x} \cdot dx = -2 \int \frac{1 - \cos^2 2x}{\cos 2x} \cdot dx \\
 &= -2 \left[ \int \frac{1}{\cos 2x} \cdot dx - \int \frac{\cos 2x}{\cos 2x} \cdot dx \right] \\
 &= -2 \left[ \int \sec 2x \cdot dx - \int \cos 2x \cdot dx \right] \\
 &= -2 \left[ \log(\sec 2x + \tan 2x) - \frac{1}{2} \sin 2x \right]
 \end{aligned}$$

$$A = \sin 2x - \log(\sec 2x + \tan 2x)$$

$$B = -\int \frac{R(x)}{W} y_2 dx = \frac{1}{2} \left[ \log 2 - \frac{1}{2} x^2 \right]$$

$$\begin{aligned}
 D - 2x - 2 &= \frac{1}{2} \int \frac{4 \tan 2x \cdot \cos 2x}{\cos 2x} dx \\
 &= \int 2 \frac{\sin 2x}{\cos 2x} \cdot \cos 2x \cdot dx \\
 &= 2 \int \sin 2x \cdot dx = 2(-\frac{1}{2} \cos 2x) = -\cos 2x
 \end{aligned}$$

$$B = -\frac{\cos 2x}{2} = -\frac{1}{2} \cos 2x$$

$$\begin{aligned}
 P.D &= Ay_1 + By_2 \\
 &= (\sin 2x - \log(\sec 2x + \tan 2x)) \cos 2x - (\cos 2x) \\
 &= (\sin 2x - \log(\sec 2x + \tan 2x)) \cos 2x - (\cos 2x)(\sin 2x)
 \end{aligned}$$

$$y = C.F + P.I$$

$$y = (\cos 2x + \sin 2x + \sin 2x - \log(\sec 2x + \tan 2x)) \cos 2x - (\cos 2x)(\sin 2x)$$

Example : 2

$$P.D^2 + 1)y = \sec x$$

Soln: Given  $y_1 = \sec x$

$$CD^2 + 1)y = \sec x$$

$$\begin{aligned}
 \text{Then } A - B &= 0 \quad m^2 + 1 = 0 \quad \text{or if } \alpha = 0 \\
 \therefore m^2 &= -1 \\
 \therefore m &= \pm i
 \end{aligned}$$



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$$\begin{aligned}
 C.F. &= e^{0x} [A \cos x + B \sin x] \\
 P.D. &= Qf_1 + Qf_2 \quad A y_1 + B y_2 \\
 A.R.D. &= - \int \frac{R(x)}{W} y_2 dx \\
 W &= \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix} \quad f_1 = \cos x, \quad f_2 = \sin x \\
 &\quad f_1' = -\sin x, \quad f_2' = \cos x \\
 W &= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1 \\
 A.R.D. &= - \int \frac{\sec x}{1} \sin x dx \\
 &= - \int \frac{\sin x}{\cos x} dx \quad \sec x/3 \\
 &= \int \tan x dx = 3 \\
 P.D. &= \log(\sec x) + C \\
 Q(x) &= \int x^n dx = \frac{x^{n+1}}{n+1} = \frac{x^1}{1} \\
 Q &= \int dx = x \\
 P.D. &= P f_1 + Q f_2 \\
 &= -\log(\sec x) \cos x + x \sin x \\
 Y &= C.F. + P.D. \\
 &= A \cos x + B \sin x - \log(\sec x) \cos x + x \sin x.
 \end{aligned}$$

(Ex) Example : 3  
 $D(D^2 + 1)y = \cos ec x$ .  
 Soln: Given,  $D(D^2 + 1)y = \cos ec x$ .  
 The A.E is  $m^2 + 1 = 0$   
 $m = \pm i$   
 $\therefore C.F. = e^{0x} [A \cos x + B \sin x]$   
 $P.D. = A y_1 + B y_2$   
 $A = - \int \frac{R(x)}{W} y_2 dx$   
 $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$   
 $= \cos^2 x + \sin^2 x = 1$



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UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Method of variation of parameter

$$A = - \int \frac{\cos x}{1} \cos x dx$$
$$= - \int \frac{1}{\sin x} \cos x dx$$

$$= - \int \cot x dx$$

$$= - \log(\sin x)$$

$$B = \int \frac{R(x)}{W} y_1 dx$$
$$= \int \frac{\cos x}{1} \sin x dx$$
$$= \int \frac{1}{\sin x} \sin x dx$$
$$= \int dx$$

$$B = x$$

$$P.D = Ay_1 + By_2$$

$$= - \log(\sin x) \cos x + x \sin x$$

$$I.F = e^{\int P.D dx} = e^{\int (-\log(\sin x)) dx}$$

$$= e^{\cos x} [A \cos x + B \sin x] - \log(\sin x)$$