



Binomial Expansion:

$$1) (1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$$

$$2) (1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$$

$$3) (1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$$

$$4) (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

Method of Variation of Parameters:

The Second order linear differential equation is

$$\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = R.$$

Step 1: Find Complementary function. From this calculate

y_1 and y_2 (coefficient of constant).

Step 2: Find Wronskian

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

where

$$P = -\int \frac{R}{W} y_2 dx$$

$$Q = \int \frac{R}{W} y_1 dx$$

$$P.I = Py_1 + Qy_2$$



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$$1) (D^2 + 4)y = \sec 2x.$$

Sol: The auxiliary equation is $m^2 + 4 = 0$

$$m^2 = -4$$

$$m = \pm 2i$$

The roots are imaginary:

$$C.F = e^{0x} [A \cos 2x + B \sin 2x]$$

$$f_1 = \cos 2x \quad f_2 = \sin 2x$$

$$f_1' = -2 \sin 2x \quad f_2' = 2 \cos 2x$$

$$W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \sin 2x & 2 \cos 2x \end{vmatrix}$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$= 2(\cos^2 2x + \sin^2 2x)$$

$$W = 2$$

$$P.I = P f_1 + Q f_2$$

$$P = - \int \frac{f_2 x}{W} dx$$

$$= - \int \frac{\sin 2x \sec 2x}{2} dx$$

$$= - \frac{1}{2} \int \sin 2x \cdot \frac{1}{\cos 2x} dx$$

$$Q = \int \frac{f_1 x}{W} dx$$

$$= \int \frac{\cos 2x \sec 2x}{2} dx$$

$$= \frac{1}{2} \int \cos 2x \cdot \frac{1}{\cos 2x} dx$$



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$$= \frac{-1}{2} \int \tan 2x \, dx$$

$$= \frac{-1}{2} \frac{\log(\sec 2x)}{2}$$

$$= \frac{-1}{4} \log \sec 2x$$

$$= \frac{1}{2} \int dx$$

$$= \frac{1}{2} (x)$$

$$= \frac{x}{2}$$

$$P.I = P.f_1 + Q.f_2$$

$$= \frac{-1}{4} \log(\sec 2x) \cos 2x + \frac{x}{2} \sin 2x$$

$$y = C.F + P.I$$

$$= A \cos 2x + B \sin 2x - \frac{1}{4} \log(\sec 2x) \cos 2x + \frac{x}{2} \sin 2x$$

2) Solve: $(D^2 + 1)y = \operatorname{cosec} x$.

The auxiliary equation is $m^2 + 1 = 0$

$$m = \pm i$$

The roots are imaginary.

$$C.F = e^{0x} [A \cos x + B \sin x]$$

$$= A \cos x + B \sin x$$

$$f_1 = \cos x$$

$$f_2 = \sin x$$

$$f_1' = -\sin x$$

$$f_2' = \cos x$$

$$W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$$



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$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$
$$= \cos^2 x + \sin^2 x$$
$$= 1$$
$$P.I = P f_1 + Q f_2$$
$$P = - \int \frac{f_2 x}{W} dx$$
$$= - \int \frac{\sin x \operatorname{cosec} x}{1} dx$$
$$= - \int \sin x \cdot \frac{1}{\sin x} dx$$
$$= - \int dx$$
$$= -x$$
$$Q = \int \frac{f_1 x}{W} dx$$
$$= \int \frac{\cos x \operatorname{cosec} x}{1} dx$$
$$= \int \cos x \cdot \frac{1}{\sin x} dx$$
$$= \int \cot x dx$$
$$= \log \sin x$$
$$P.I = -x \cos x + \log(\sin x) \sin x$$
$$y = C.F. + P.I$$
$$= -A \cos x + B \sin x - x \cos x + \sin x \log \sin x$$

$$y = -A \cos x + B \sin x - x \cos x + \log(\sin x) \sin x$$



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$$3) (D+1)y = \cot x$$

Sol: The auxiliary equation is $m^2 + 1 = 0$

$$m^2 = -1$$

$$m = \pm i$$

The roots are imaginary.

$$C.F = e^{0x} [A \cos x + B \sin x]$$

$$= A \cos x + B \sin x$$

$$f_1 = \cos x$$

$$f_2 = \sin x$$

$$f_1' = -\sin x$$

$$f_2' = \cos x$$

$$W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x$$

$$= 1$$

$$P.I = P f_1 + Q f_2$$

$$P = - \int \frac{f_2 x}{W} dx$$

$$= - \int \frac{\sin x \cot x}{1} dx$$

$$= - \int \sin x \times \frac{\cos x}{\sin x} dx$$

$$= - \int \cos x dx$$

$$= - \sin x$$

$$P = - \sin x$$

$$Q = \int \frac{f_1 x}{W} dx$$

$$= \int \frac{\cos x \cot x}{1} dx$$

$$= \int \frac{\cos^2 x}{\sin x} dx$$

$$= \int \frac{1 - \sin^2 x}{\sin x} dx$$

$$= \int \left(\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} \right) dx$$

$$= \int \operatorname{cosec} x dx - \int \sin x dx$$

$$= -\log (\operatorname{cosec} x + \cot x) - (-\cos x)$$



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$$P.I = -\sin x \cos x + [-\log(\operatorname{cosec} x + \cot x) + \cos x] \sin x$$

$$y = C.F + P.I$$

$$y = A \cos x + B \sin x - \sin x \cos x + [-\log(\operatorname{cosec} x + \cot x) + \cos x] \sin x$$

4) Solve: $(D^2 + 4)y = \tan 2x$.

Sol: The auxiliary equation is

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$C.F = e^{0x} [A \cos 2x + B \sin 2x]$$

$$= A \cos 2x + B \sin 2x$$

$f_1 = \cos 2x$ $f_2 = -\sin x$
 $f_1' = -2 \sin 2x$ $f_2' = 2 \cos 2x$

$$W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \sin 2x & 2 \cos 2x \end{vmatrix}$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$= 2(\cos^2 2x + \sin^2 2x)$$

$$= 2(1)$$

$$W = 2$$



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$$\begin{aligned} P.I &= P f_1 + Q f_2 \\ P &= - \int \frac{f_2 x}{w} dx & Q &= \int \frac{f_1 x}{w} dx \\ &= - \int \frac{\sin 2x \tan 2x}{2} dx & &= \int \frac{\cos 2x \tan 2x}{2} dx \\ &= - \frac{1}{2} \int \frac{\sin^2 2x}{\cos 2x} dx & &= \int \cos 2x \frac{\sin 2x}{\cos 2x} dx \\ &= - \frac{1}{2} \int \frac{1 - \cos^2 2x}{\cos 2x} dx & &= \int \sin 2x dx \\ &= - \frac{1}{2} \int \frac{1}{\cos 2x} dx - \frac{\cos^2 2x}{\cos 2x} dx & &= - \frac{\cos 2x}{2} \\ &= - \frac{1}{2} \int \sec 2x dx - \cos 2x dx \\ &= - \frac{1}{2} \left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2} \right] \end{aligned}$$

$$\begin{aligned} P.I &= P f_1 + Q f_2 \\ &= - \frac{1}{2} \left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2} \right] \cos 2x \\ &\quad - \frac{\cos 2x \sin 2x}{2} \end{aligned}$$

$$\begin{aligned} y &= C.F + P.I \\ y &= A \cos 2x + B \sin 2x - \frac{1}{2} \left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2} \right] \cos 2x - \frac{\cos 2x \sin 2x}{2} \end{aligned}$$