



Legendre's Linear Differential Equations:

$$(ax+b)^2 y'' + (ax+b)y' + y = Q(x)$$

$$\text{Let } e^z = (ax+b)$$

$$z = \log(ax+b)$$

$$(ax+b)D = a\theta$$

$$(ax+b)^2 D^2 = a^2(\theta^2 - \theta)$$

1. Transform the equation to construct coefficient

$$(2x+3)^2 y'' - (2x+3)y' + 2y = 6x$$

$$(2x+3)^2 D^2 y - (2x+3)Dy + 2y = 6x$$

$$[(2x+3)^2 D^2 - (2x+3)D + 2]y = 6x \rightarrow \textcircled{1}$$

$$e^z = 2x+3$$

$$2x = e^z - 3$$

$$x = \frac{e^z - 3}{2}$$

$$z = \log(2x+3)$$

$$(2x+3)D = 2\theta$$

$$(2x+3)^2 D^2 = 2^2(\theta^2 - \theta)$$

$$= 4(\theta^2 - \theta)$$

Sub in equ $\textcircled{1}$

$$(4(\theta^2 - \theta) - 2\theta + 2)y = 6\left(\frac{e^z - 3}{2}\right)$$

$$(4\theta^2 - 4\theta - 2\theta + 2)y = 3e^z - 9$$

$$(4\theta^2 - 6\theta + 2)y = 3e^z - 9$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



$$2) \text{ Solve: } [(x+2)^2 D^2 - (x+2)D + 1]y = 3x+4.$$

$$\text{Sol:- } [(x+2)^2 D^2 + (x+2)D + 1]y = 3x+4 \rightarrow (1)$$

$$e^z = x+2$$

$$x = e^z - 2$$

$$z = \log(x+2)$$

$$(x+2)D = \theta$$

$$(x+2)D^2 = 1^{\theta}(\theta^2 - \theta) = \theta^2 - \theta$$

Sub in equ (1)

$$[(\theta^2 - \theta) - \theta + 1]y = 3[e^z - 2] + 4$$

$$(\theta^2 - \theta - \theta + 1)y = 3e^z - 6 + 4$$

$$(\theta^2 - 2\theta + 1)y = 3e^z - 2 \rightarrow (2)$$

C.F The auxiliary equation is $m^2 - 2m + 1 = 0$

$$(m-1)(m-1) = 0$$

$$m = 1, 1$$

$$\text{C.F} = (A+Bz)e^z$$

$$\text{P.I}_1 = \frac{1}{\theta^2 - 2\theta + 1} 3e^z$$

$$= \frac{1}{1^2 - 2(1) + 1} 3e^z$$

$$= \frac{1}{0} 3e^z$$

$$\text{P.I}_1 = \frac{z}{2\theta - 2} 3e^z = \frac{z}{2(1) - 2} 3e^z = \frac{z}{0} 3e^z = \frac{z}{2} 3e^z$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



$$P.I_2 = \frac{1}{\theta^2 - 2\theta + 1} (-2e^{0z})$$

$$= \frac{1}{\theta^2 - 2(0) + 1} (-2(1))$$

$$= \frac{1}{1} (-2)$$

$$= -2$$

$$y = C.F + P.I_1 + P.I_2$$

$$= (A+Bz)e^z + \frac{z}{2} 3e^z - 2$$

$$= [A + B \log(x+2)] e^{\log(x+2)} + \frac{3 \log(x+2)^2}{2} e^{\log(x+2)} - 2$$

$$= [A + B \log(x+2)(x+2) + \frac{3 \log(x+2)^2}{2} (x+2) - 2]$$

$$y = [A + B \log(x+2)(x+2) + \frac{3 \log(x+2)^2}{2} (x+2) - 2]$$