3) $(D^2+1)y = \cot x$

Sol: The auxiliary equation is $m^2+1=0$

$m^2 = -1$

$m = \pm i$

The roots are imaginary.

$C.F = e^{0x}[A\cos x + B\sin x]$

$= A\cos x + B\sin x$

$f_1 = \cos x \qquad f_2 = \sin x$

$f_1' = -\sin x \qquad f_2' = \cos x$

$$W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$$

$= \cos^2 x + \sin^2 x$

$= 1$

$P.I = P f_1 + Q f_2$

$P = -\int \frac{f_2 X}{W} dx$

$= -\int \frac{\sin x \cot x}{1} dx$

$= -\int \sin x \times \frac{\cos x}{\sin x} dx$

$= -\int \cos x \, dx$

$= -\sin x$

$P = -\sin x$

$Q = \int \frac{f_1 X}{W} dx$

$= \int \frac{\cos x \cot x}{1} dx$

$= \int \frac{\cos^2 x}{\sin x} dx$

$= \int \frac{1-\sin^2 x}{\sin x} dx$

$= \int \left(\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}\right) dx$

$= \int \operatorname{cosec} x \, dx - \int \sin x \, dx$

$= -\log(\operatorname{cosec} x + \cot x) - (-\cos x)$

$$P.I = -\sin x \cos x + [-\log(\operatorname{cosec} x + \cot x) + \cos x]\sin x$$

$$y = C.F + P.I$$

$$y = A\cos x + B\sin x - \sin x \cos x + [-\log(\operatorname{cosec} x + \cot x) + \cos x]\sin x.$$

4) Solve: $(D^2+4)y = \tan 2x$.

Sol:- The auxiliary equation is

$$m^2+4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$C.F = e^{0x}[A\cos 2x + B\sin 2x]$$

$$= A\cos 2x + B\sin 2x$$

$f_1 = \cos 2x$ $\quad$ $f_2 = \sin 2x$

$f_1' = -2\sin 2x$ $\quad$ $f_2' = 2\cos 2x$

$$W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$= 2\cos^2 2x + 2\sin^2 2x$$

$$= 2(\cos^2 2x + \sin^2 2x)$$

$$= 2(1)$$

$$W = 2$$

$P.I = Pf_1 + Qf_2$

$P = -\int \frac{f_2 x}{W} dx$

$= -\int \frac{\sin 2x \tan 2x}{2} dx$

$= -\frac{1}{2}\int \frac{\sin^2 2x}{\cos 2x} dx$

$= -\frac{1}{2}\int \frac{1-\cos^2 2x}{\cos 2x} dx$

$= -\frac{1}{2}\int \frac{1}{\cos 2x} dx - \frac{\cos^2 2x}{\cos 2x} dx$

$= -\frac{1}{2}\int \sec 2x\, dx - \cos 2x\, dx$

$= -\frac{1}{2}\left[\left[\frac{\log(\sec 2x + \tan 2x)}{2}\right] - \frac{\sin 2x}{2}\right]$

$Q = \int \frac{f_1 x}{W} dx$

$= \int \frac{\cos 2x \tan 2x}{2} dx$

$= \int \cos 2x \frac{\sin 2x}{\cos 2x} dx$

$= \int \sin 2x\, dx$

$= -\frac{\cos 2x}{2}$

$P.I = Pf_1 + Qf_2$

$= -\frac{1}{2}\left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right]\cos 2x$

$- \frac{\cos 2x}{2} \sin 2x.$

$y = C.F + P.I$

$y = A\cos 2x + B\sin 2x - \frac{1}{2}\left[\frac{\log(\sec 2x + \tan 2x)}{2}\right.$

$\left. - \frac{\sin 2x}{2}\right]\cos 2x - \frac{\cos 2x}{2} \sin 2x.$