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UNIT III – FINITE ELEMENT TECHNIQUES

Introduction to Finite Element Methods

The finite element method (FEM) is a numerical technique for solving partial differential equations (PDEs). Its first essential characteristic is that the continuum field, or domain, is subdivided into cells, called elements, which form a grid. The elements (in 2D) have a triangular or quadrilateral form and can be rectilinear or curved. The grid itself need not be structured. With unstructured grids and curved cells, complex geometries can be handled with ease. This important advantage of the method is not shared by the finite difference method (FDM) which needs a structured grid, which, however, can be curved. The finite volume method (FVM), on the other hand, has the same geometric flexibility as the FEM.

The second essential characteristic of the FEM is that the solution of the discrete problem is assumed a priori to have a prescribed form. The solution has to belong to a function space, which is built by varying function values in a given way, for instance linearly or quadratically, between values in nodal points. The nodal points, or nodes, are typical points of the elements such as vertices, mid-side points, mid-element points, etc. Due to this choice, the representation of the solution is strongly linked to the geometric representation of the domain. This link is, for instance, not as strong in the FVM.

The third essential characteristic is that a FEM does not look for the solution of the PDE itself, but looks for a solution of an integral form of the PDE. The most general integral form is obtained from a weighted residual formulation. By this formulation, the method acquires the ability to naturally incorporate differential type boundary conditions and allows easily the construction of higher-order accurate methods. The ease in obtaining higher order accuracy and the ease of implementation of boundary conditions form a second important advantage of the FEM. To accuracy, the FEM is superior to the FVM, where higher-order accurate formulations are quite complicated.

The combination of the representation of the solution in a given function space, with the integral formulation treating rigorously the boundary conditions, gives the method an extremely strong and rigorous mathematical foundation.

A final essential characteristic of the FEM is the modular way in which the discretization is obtained. The discrete equations are constructed from contributions on the element level which afterwards are assembled.

Historically, the finite element method originates from the field of structural mechanics. This has some remnants in the terminology. In structural mechanics, the partial differential formulation of a problem can be replaced by an equivalent variational formulation, i.e. the minimization of an energy integral over the domain.

The variational formulation is a natural integral formulation for the FEM. In fluid dynamics, in general, a variational formulation is not possible. This makes it less obvious how to formulate a finite element method. The history of computational fluid dynamics (CFD) shows that every essential break-through has first been made in the context of the finite difference method or the finite volume method and that it always has taken considerable time, often much more than a decade, to incorporate the same idea into the finite element method. The history of CFD, on the other hand, also shows that, once a suitable FEM formulation has been found, the FEM is almost exclusively used. This is due to the advantages of the treatment of complex geometries and obtaining higher-order accuracy.

The development of the finite element method in fluid dynamics is at present still far from ended. For the simplest problems such as potential flows, both compressible and incompressible, and incompressible Navier-Stokes flows at low Reynolds numbers, the finite element method is more or less full-grown, although new evolutions, certainly for Navier-Stokes problems, are continuing. More complex problems like compressible flows governed by Euler- or Navier-Stokes equations or incompressible viscous flows at high Reynolds numbers still form an area of active research.

In this introductory text, the option is taken to explain the basic ingredients of the finite element method on a very simple, purely mathematical, problem and to give fluid dynamics illustrations in detail only for simple problems. For more complex problems, only a basic description is given about further literature. Also, in the explanation of the method, all mathematical aspects are systematically avoided. For the mathematical aspects, reference is made to further literature. This makes the text accessible for a reader with almost no knowledge of functional analysis and numerical analysis. For the fluid dynamics illustrations, the option has been taken to use only simple techniques, so that the detailed examples can be reproduced by the reader not familiar with general computational fluid dynamics or even general fluid dynamics. This text therefore is to be seen as the absolute minimum introduction to the subject.

The text is in no way complete and the author deliberately has taken the risk to be seen as naïve by a more informed reader. A reference list is given for a deeper introduction. A reader beginning with computational fluid dynamics should be aware that a complete study of the finite element method may take considerable time and may necessitate, depending on background, a considerable effort. The method is much less intuitive than the finite difference method and the finite volume method and requires a more fundamental attitude for mathematical formulations.

This introductory text therefore is also meant to create some enthusiasm for the method by showing its power with simple examples and to justify in this way the need for further study. It is the conviction of the author that a practitioner of CFD, even if it is not his or her intention to use the FEM, should have at least a basic knowledge of the method. This is in particular useful to the treatment of boundary conditions. Also, one should consider that the impact of the FEM in CFD is already extremely important and that it probably will grow in the future.