DEPARTMENT OF AEROSPACE ENGINEERING

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UNIT III – FINITE ELEMENT TECHNIQUES

Variational Formulation and Shape Functions

1. Variational Formulation:

- **From Strong to Weak Forms:**
  - Many physical problems are described by differential equations, known as the "strong form." These can be difficult to solve directly, especially for complex geometries.
  - Variational formulation transforms the strong form into a "weak form" using techniques like the Galerkin method or the principle of virtual work.
  - The weak form involves integrals and relaxes the continuity requirements of the solution, making it easier to find approximate solutions.
  - Essentially, it converts the problem into finding a function that minimizes a certain function or satisfies an integral equation.
- **Key Idea:**
  - Instead of requiring the solution to satisfy the differential equation at every point, the variational formulation requires it to satisfy an integral statement over the domain. This allows for approximate solutions that may not be perfectly smooth.

2. Shape Functions:

- **Approximation within Elements:**
  - In FEM, the domain is divided into small elements. Within each element, the unknown solution is approximated using polynomial functions. These polynomial functions are called "shape functions."
  - Shape functions define the variation of the solution within an element in terms of the nodal values (the values of the solution at the element's nodes).
- **Properties:**
  - **Nodal Property:** A shape function associated with a particular node has a value of 1 at that node and 0 at all other nodes.
  - **Completeness:** The shape functions must be able to represent constant and linear fields.
  - **Continuity:** Shape functions are designed to ensure the continuity of the solution across element boundaries.
  - **Purpose:**
    - They provide a way to interpolate the solution within an element based on the nodal values.
    - They are crucial for assembling the element equations into a global system of equations.
- **Example:**
  - In a simple 1D linear element, the shape functions are linear polynomials. They allow you to approximate the solution at any point within the element as a linear combination of the nodal values.

In essence:

- Variational formulation provides a mathematical framework for converting differential equations into a form suitable for numerical solutions.
- Shape functions are the building blocks for approximating the solution within each element, enabling the discretization of the problem.

These two concepts work together to form the basis of the finite element method, allowing for the numerical solution of complex engineering and scientific problems.