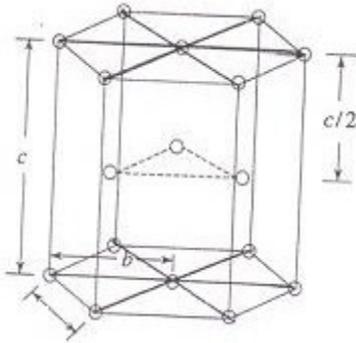




HEXAGONAL CLOSED PACKED STRUCTURE



- It consists of three layers of atoms.
- The bottom layer has six corner atoms and one face centred atom.
- The middle layer has three full atoms.
- The upper layer has six corner atoms and one face centred atom.
- Each and every corner atom contributes $1/6$ of its part to one unit cell.
- The number of total atoms contributed by the corner atoms of both top and bottom layers is $1/6 \times 12 = 2$.
- The face centred atom contributes $1/2$ of its part to one unit cell.
- Since there are 2 face centred atoms, one in the top and the other in the bottom layers, the number of atoms contributed by face centred atoms is $1/2 \times 2 = 1$.
- Besides these atoms, there are 3 full atoms in the middle layer.
- Total number of atoms present in an HCP unit cell is $2+1+3 = 6$.

CO-ORDINATION NUMBER (CN)

- The face centered atom touches 6 corner atoms in its plane.
- The middle layer has 3 atoms.



- There are three more atoms, which are in the middle layer of the unit cell.
- Therefore the total number of nearest neighbours is $6+3+3=12$.

ATOMIC RADIUS (R)

- Consider any two corner atoms.
- Each and every corner atom touches each other. Therefore $a = 2r$.

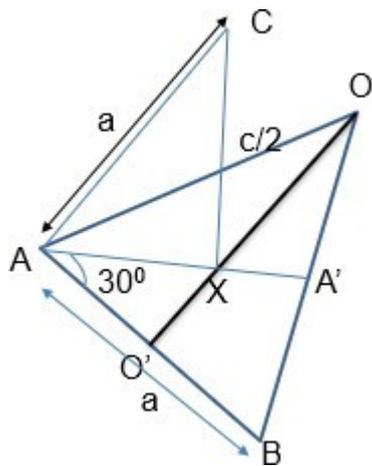
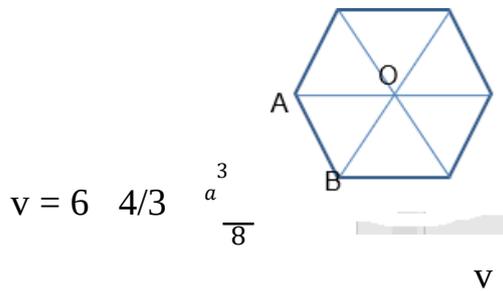
i.e., The atomic radius, $r = a/2$

ATOMIC PACKING FACTOR (APF)

$$APF = v/V$$

$$v = 6 \cdot \frac{4}{3} r^3$$

Substitute $r = a/2$



$AB = AC = BO = 'a'$. $CX = c/2$ where c height of the hcp unit cell.

Area of the base = $6 \cdot \text{area of the triangle } ABO = 6 \cdot \frac{1}{2} AB \cdot OO$

Area of the base = $6 \cdot \frac{1}{2} a \cdot OO$

In triangle $O'OB$

$$\cos 30^\circ = \frac{OO'}{BO} = \frac{OO'}{a}$$

$$OO = a \cos 30^\circ = a \cdot \frac{\sqrt{3}}{2}$$

Now, substituting the value of OO ,

$$\frac{\sqrt{3}}{2}$$

$$\frac{3\sqrt{3}a^2}{8}$$



Area of the base = $6 \frac{1}{2} a$
 $V = \text{Area of the base} \times \text{height}$

$$V = \frac{3\sqrt{3}a^2}{2} \times c$$
$$\therefore \text{APF} = \frac{v}{V} = \frac{\pi a^3}{\frac{3\sqrt{3} a^2 c}{2}}$$
$$\therefore \text{APF} = \frac{2\pi a^3}{3\sqrt{3}a^2 c} = \frac{2\pi}{3\sqrt{3}} \frac{a}{c}$$

Determination of c/a ratio:

In the triangle ABA ,

$$\cos 30^\circ = \frac{A'}{AB}$$

30



$$AA = AB \cos 30^\circ = a \frac{3}{2}$$

$$\text{But } AX = \frac{2}{3} AA = \frac{2}{3} \cdot a \frac{\sqrt{3}}{2}$$

$$\text{i.e. } AX = \frac{a}{\sqrt{3}}$$

In the triangle

$$AXC, AC^2 =$$

$$AX^2 + CX^2$$

Substituting the values of AC, AX and CX,

$$a^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + \left(\frac{c}{2}\right)^2$$

$$a^2 = \frac{a^2}{3} + \frac{c^2}{4}$$

$$\frac{c^2}{4} = a^2 - \frac{a^2}{3}$$

$$\frac{c^2}{4} = a^2 \left(1 - \frac{1}{3}\right)$$

$$\frac{c^2}{a^2} = \frac{8}{3}$$

$$\frac{c}{a} = \sqrt{\frac{8}{3}}$$

Now substituting the value of c/a to calculate APF of an hcp unit cell,

$$\begin{aligned} \text{APF} &= \frac{2\pi}{3\sqrt{3}} \sqrt{\frac{3}{8}} \\ &= \frac{2\pi}{3\sqrt{3}} \frac{\sqrt{3}}{2\sqrt{2}} \end{aligned}$$

$$\therefore \text{APF} = \frac{\pi}{3\sqrt{2}} = 0.74$$