## UNIT-3

# Free Electron theory of metals

**Classical free electron theory is based on the following postulates:**

1. A solid metal is composed of atoms and the atoms have nucleus, around which there are revolving electrons.

2. In a metal the valance electrons of atoms are free to move throughout the volume of the metal like gas molecules of a perfect gas in a container

3. The free electrons move in a random directions and collide with either positive ions fixed to the lattice or other free electrons and collisions are elastic in nature i.e. there is no loss of energy.

4. The movement of free electrons obeys the classical kinetic theory of gasses. The mean K.E. of a free electron is equal to that of gas molecule $\left(\frac{3}{2}KT\right)$.

5. The electron velocities in a metal obey Maxwell-Boltzman distribution of velocities.

6. The free electrons move in a uniform potential field due to ions fixed in the lattice

7. When an electric field is applied to the metal the free electrons are accelerated. The accelerated electrons move in opposite direction of the applied.

8. The electric conduction is due to the free electrons only.

## ROOT MEAN SQUARE (R.M.S.) VELOCITY:

Let $\overline{C}$ be the r.m.s velocity of the free electron. then the

$$Kinetic\ energy = \frac{1}{2} m\overline{C}^2$$

But according to the classical free electron theory the mean

$$\text{Kinetic Energy} = \left(\frac{3}{2}KT\right).$$

$$\therefore \frac{1}{2}m\overline{c}^2 = \frac{3}{2}KT$$

$$\Rightarrow \boxed{\overline{c} = \sqrt{\frac{3KT}{m}}}$$ where $\overline{c}$ = root mean square velocity

## MEAN FREE PATH $(\lambda)$ AND MEAN COLLISION TIME $(\tau_C)$

The average distance travelled by an electron between two successive collisions in the presence of applied filed is known as 'Mean free path $(\lambda)$'.

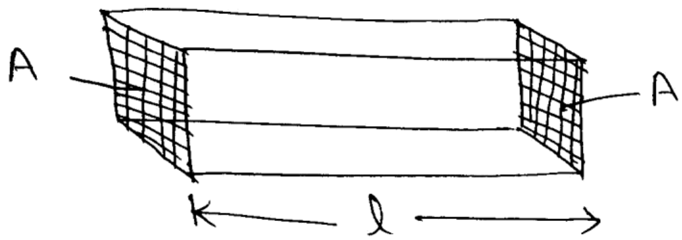
The time taken by an electron between two successive collisions is known as "Mean Collision Time $(\tau_C)$" of the electron

$$\boxed{\tau_c = \frac{\lambda}{\overline{c}} = \lambda\sqrt{\frac{m}{3KT}}}$$

## DRIFT VELOCITY ($v_d$):

It is the average velocity acquired by the free electrons of a metal in a particular direction during the application of the electric field.

## ELECTRICAL CONDUCTVITY IN METALS:




Let us consider a conductor of length $l$ and area of cross section $A$

The volume of the conductor $= Al$

If there are $n$ number of electrons per unit volume of the metal

then the total number of electrons in the metal $= Aln$

If $e$ is the charge of the electron then the total charge $q$ due to all electrons in the conductor is given by $q = Aln.e$

Let $t$ be the time taken by the electron to move from one end to other end then

$$\text{Current}\,(I) = \frac{charge}{time} = \frac{q}{t} = \frac{Alne}{t}$$

$$\text{But } \frac{l}{t} = v_d$$

$$\therefore I = Anev_d$$

$$\Rightarrow v_d = \frac{I}{Ane} = \frac{J}{ne}$$

Where $J =$ current density $= \frac{I}{A}$

In a metal the current density $J$ is given bythe equation

$$J = nev_d \text{-----------------}(1)$$

Where $n =$ number of electrons per Unit volume, $e =$ electron charge and $v_d =$ drift velocity

If $E$ is the applied electric field then the electric force acting on a free electron is given by

$$F = eE \text{..............................}(2)$$

From Newton's IInd law $F = ma$ -------------------------------- (3)

From (2) and (3) $ma = eE$

i.e. $a = \frac{eE}{m}$

but $a$ =drift velocity/collision time $= \frac{v_d}{\tau_c}$

$$v_d = a\tau_c = \frac{eE}{m}\tau_c$$

$$\therefore J = ne.\frac{eE}{m}\tau_c = \frac{ne^2E}{m}\tau_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

But from microscopic form of ohms law

$$J = \sigma E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

On comparing Eq(4)&(5)

$\therefore$ Conductivity $\sigma = \frac{ne^2}{m}\tau_c$ or Re *sistivity*.$\rho = \frac{m}{ne^2\tau_c}$

Conductivity may also be expressed in terms of mobility ( $\mu$ ) which is defined as drift velocity per unit electric field

$$\mu = \frac{v_d}{E} = \frac{e}{m}\tau_c$$

From (4) $\boxed{\sigma = ne\mu}$

## *RELAXATION TIME*($\tau_r$)

Under the influence of an external electric field free electrons attain a directional velocity of motion. If the field is switched off the velocity starts decreasing exponentially. Such a process that tends to restore equilibrium is called relaxation process.

If $v_o$ is the velocity at $t = 0$ at which the field is switched off.

The velocity at any time is given by

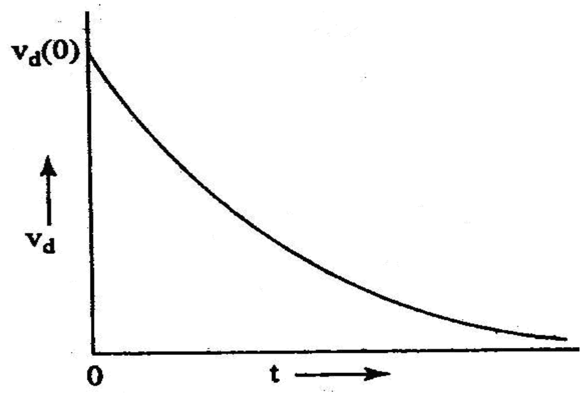
$$v = v_o e^{\frac{-t}{\tau_r}}$$

In the above expression $\tau_r$ = relaxation time

If $t = \tau_r$

$$v = v_o e^{\frac{-t}{\tau_r}} = v_o e^{-1} = \frac{v_o}{e}$$

$\therefore$ Relaxation time $\tau_r$ is defined as the time required for the electron to reduce its velocity to $\frac{1}{e}$ of its initial value. (OR) time taken for the drift velocity to decay $\frac{1}{e}$ of its initial value.




## Failure of classical free electron theory:

1. The phenomena such as photo electric effect, Compton Effect and black body radiation could not be explained by classical free electro theory.
2. According to classical theory the value of specific heat of metals is given by 4.5R (R =Universal gas constant) where as the experimental value is nearly 3R(Dulang Petit law)
3. Electrical conductivity of semiconductor or insulator could not be explained by using this model.
4. According to classical free electron model $\frac{K}{\sigma T}$ is constant.(Widemann-franz law) as this not constant at low temperatures.
5. Ferromagnetism could not be explained by this theory
6. According to classical free electron theory,
   Resistivity

   $$\rho = \frac{m}{ne^2\tau_c} = \frac{m}{ne^2}\sqrt{\frac{3KT}{m}}\frac{1}{\lambda} = \frac{\sqrt{3KTm}}{ne^2\lambda}$$

   $$\rho = \sqrt{T}$$

   But according to experiments $\rho \alpha T$

## QUANTUM FREE ELECTRON THEORY:

Somerfield applied quantum mechanics to explain conductivity phenomenon in metals. He has improved the Drude- Lorentz theory by quantizing the free electron energy and retaining the classical concept of force motion of electrons at random.

**ASSUMPTIONS**

1. The electrons are free to move with in the metal like gaseous molecules. They are confined to the metal due to surface potential.

2. The velocities of electrons obey Fermi-Dirac distribution because electrons are spin – half particles.

3. The electrons would go into different energy levels and obey Pauli's exclusion principle.