



SNS COLLEGE OF TECHNOLOGY

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COIMBATORE-641 035, TAMIL NADU



THE VARIATION OF THE FERMI LEVEL WITH TEMPERATURE

To derive the variation of the Fermi level with temperature, especially in semiconductors, we need to consider how the electron and hole concentrations are distributed according to Fermi-Dirac statistics. Here's the derivation, focusing on intrinsic semiconductors and extending the concept to extrinsic (doped) semiconductors.

1. Fermi-Dirac Distribution:

The probability $f(E)$ that an energy state E is occupied by an electron at temperature T is given by the Fermi-Dirac distribution function:

$$f(E) = \frac{1}{1 + \exp \frac{E - E_F}{k_B T}}$$

Where:

- E is the energy of the electron state.
- E_F is the Fermi energy (Fermi level).
- k_B is Boltzmann's constant.
- T is the absolute temperature.

At absolute zero temperature, the Fermi level is the highest occupied energy, but at higher temperatures, this distribution smears out, and electrons are excited to higher energy states.

2. Electron and Hole Concentrations:

In semiconductors, the electron concentration n in the conduction band and the hole concentration p in the valence band are determined by integrating the product of the density of states $g(E)$ and the probability $f(E)$ for electrons (or $1 - f(E)$ for holes).

a) Electron concentration n in the conduction band:

The electron concentration in the conduction band is given by:

$$n = \int_{E_c}^{\infty} g_c(E) f(E) dE$$

Where:

- E_c is the energy of the conduction band edge.
- $g_c(E)$ is the density of states in the conduction band.

For a semiconductor, the density of states in the conduction band is approximated as:

$$g_c(E) = A_c \sqrt{E - E_c}$$

Thus, the electron concentration becomes:

$$n = A_c \int_{E_c}^{\infty} \sqrt{E - E_c} \frac{1}{1 + \exp \frac{E - E_F}{k_B T}} dE$$

This integral is simplified using the Maxwell-Boltzmann approximation (valid for non-degenerate semiconductors at high temperatures), where the Fermi level is much lower than the conduction band edge, so $\exp \frac{E - E_F}{k_B T} \gg 1$. The electron concentration is then approximated as:

$$n \approx N_c \exp -\frac{E_c - E_F}{k_B T}$$

Where N_c is the effective density of states in the conduction band.

b) Hole concentration p in the valence band:

Similarly, the hole concentration in the valence band is given by:

$$p = \int_{-\infty}^{E_v} g_v(E)(1 - f(E))dE$$

Where:

- E_v is the energy of the valence band edge.
- $g_v(E)$ is the density of states in the valence band.

Using the Maxwell-Boltzmann approximation for the hole concentration:

$$p \approx N_v \exp \left(-\frac{E_F - E_v}{k_B T} \right)$$

Where N_v is the effective density of states in the valence band.

3. Intrinsic Carrier Concentration:

For an intrinsic semiconductor, the electron concentration n equals the hole concentration p (i.e., $n = p = n_i$), where n_i is the intrinsic carrier concentration. Using the above relations for n and p :

$$n_i = N_c \exp \left(-\frac{E_c - E_F}{k_B T} \right) = N_v \exp \left(-\frac{E_F - E_v}{k_B T} \right)$$

Taking the product of both sides:

$$n_i^2 = N_c N_v \exp \left(-\frac{E_c - E_v}{k_B T} \right)$$

This simplifies to:

$$n_i = \sqrt{N_c N_v} \exp \left(-\frac{E_g}{2k_B T} \right)$$

Where $E_g = E_c - E_v$ is the bandgap of the semiconductor.