AVERAGE SPEED AND RELATIVE SPEED

Formula: 5

When two bodies A and B are moving with speed a km/h and b km/h respectively, then the relative speed of two bodies is

(i) $(a + b)$km/h (If they are moving in opposite direction)

(ii) $(a - b)$km/h (If they are moving in same direction)

Example: 9 Two persons are moving in the directions opposite to each other. The speeds of the both persons are 5 km/h and 3 km/h, respectively. Find the relative speed of the two persons in respect of each other.

Solution: We know that, the two speeds will be added, if the motion of two objects is in opposite direction.

Required relative speed = 5+3=8 km/h.

Example: 10 Two trains are running in the same direction. The speeds of two trains are 5 km/h and 15 km/h, respectively. What will be the relative speed of second train, with respect to first?

Solution: We know that, if two trains are running in same direction, then difference in speeds is the required relative speed.

Required relative speed = $15 - 5 = 10$ km/h.

Formula: 6

When a body travels with different speeds for different durations, then average speed of that body for the complete Journey is defined as the total distance covered by the body divided by the total time taken to cover the distance.

$$\text{Average speed} = \frac{\text{Total distance covered by a body}}{\text{Total time taken by the body}}$$

Example: 11 A person covers a distance 20 km by bus in 35 min. After deboarding the bus, he took rest for 20 min and covers another 10 km by a taxi in 20 min. Find his average speed for the whole journey.

Solution: Total distance covered = (20+10) km = 30 km

Total time taken = (35 + 20 + 20)min = 75 min = $\frac{5}{4}h$

According to the formula, Average speed = $\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{30}{\frac{5}{4}} = 24$ km/h

So, the average speed of the person for the whole journey is 24 km/h.

Example: 12 If a person covers 40 km at a speed of 10 km/h by a cycle, 25 km at 5 km/h on foot and another 100 km at 50 km/h by bus. Then, find his average speed for the whole journey?

Solution : Here, total distance covered by the person = (40 + 25 + 100) km = 165 km

Time taken to cover 40 km = $\frac{40}{10} = 4\ h$

Time taken to cover 25 km = $\frac{25}{5} = 5\ h$

Time taken to cover 100 km = $\frac{100}{50} = 2\ h$

Total time taken for whole journey = (4 +5+2)h = 11 h

According to formula,

Average speed = $\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{165}{11} = 15$ km/h.

Note :

If a body covers a distance D_1 at S_1 km/h, D_2 at S_2 km/h, D_3 at S_3 km/h and so on upto D_n at S_n, then

Average speed = $\frac{D_1+D_2+D_3+D_4+D_5+\cdots\ldots\ldots+D_n}{\frac{D_1}{S_1}+\frac{D_2}{S_2}+\frac{D_3}{S_3}+\frac{D_4}{S_4}+\cdots\ldots\ldots+\frac{D_n}{S_n}}$

Average speed $\neq \frac{S_1+S_2+S_3+S_4+\cdots\ldots\ldots S_n}{n}$

Fast Track Techniques to solve the Questions:

Technique : 1

When a certain distance is covered at speed A and the same distance is covered at speed B, then the average speed during the whole journey is given by $\frac{2AB}{A+B}$.

Example: 13 Shantanu covers a certain distance by car driving at 35 km/h and he returns back to the starting point riding on a scooter with a speed of 25 km/h. Find the average speed for the whole journey.

Solution: Here, $A = 35\ km/h$ and $B = 25\ km/h$

According to the formula,

Average Speed = $\frac{2AB}{A+B} = \frac{2\times35\times25}{60} = \frac{175}{6} = 29.16\ km/h$

Technique: 2

If a body covers $\frac{1}{a}$ part of journey at $A\ km/h$, $\frac{1}{b}$ part at $B\ km/h$, $\frac{1}{c}$ part at $C\ km/h$ and soon, then the average speed of body for the whole journey is $\frac{1}{\frac{1}{aA}+\frac{1}{bB}+\frac{1}{cC}+\frac{1}{dD}+\cdots\ldots}$

Example: 14 A person covers half of his journey at $30\ km/h$, one-third at $40\ km/h$ and rest of his journey at $20\ km/h$. Find his average speed for the whole journey.

Solution: Here, the person cover $\frac{1}{2}$ of his journey at $30\ km/h$, $\frac{1}{3}$ at $40\ km/h$ and rest $\left(1-\frac{1}{2}-\frac{1}{3}=\frac{1}{6}\right)$ at $20\ km/h$.

Average speed for whole journey $= \frac{1}{\frac{1}{2\times30}+\frac{1}{3\times40}+\frac{1}{6\times20}}$

$$= \frac{1}{\frac{1}{60}+\frac{1}{120}+\frac{1}{120}} = \frac{1}{\frac{2+1+1}{120}} = \frac{120}{2+1+1} = 30\ \text{km/h}$$

Technique: 3

When a person covers a certain distance between two certain places with speed 'a', he gets his destination late by time t_1 but when he covers the same distance with speed 'b', he reaches his destination t_2 time earlier. In this case, the distance between two places is given by

$$D = \frac{ab(t_1 + t_2)}{b - a}$$

Example: 15 Aashutosh covers a certain distance between his home and college by cycle. Having an average speed of $30\ km/h$, he is late by 20 min. However, with a speed of $40\ km/h$, he reaches his college 10 min earlier. Find the distance between his house and college.

Solution: Here, $a = 30, b = 40$

$t_1 = \frac{20}{60}$ and $t_2 = \frac{10}{60}$

According to the formula, $D = \frac{ab(t_1+t_2)}{b-a}$

Required distance $= \frac{30\times40}{40-30} \times \left(\frac{20+10}{60}\right) = \frac{30\times40}{10} \times \frac{30}{60} = (30 \times 4) \times \frac{1}{2} = 60\ km$

Note: t_1 time late and t_2 time earlier make a difference of $(t_1 + t_2)$.

Technique: 4

When a person reaches a certain distance with speed 'a', he gets late by t_1 time and when he increases his speed by 'b' to cover the same distance, then he still gets late by t_2 time. In this case, the distance is calculated by $D = (t_1 - t_2)(a + b)\frac{a}{b}$.

Example: 16 A boy walking at a speed of $20\ km/h$ reaches his school 30 min late. Next time he increases his speed by $4\ km/h$ but still he is late by 10 min. Find the distance of the school from his home.

Solution: Here, $a = 20\ km/h$, $b = 4\ km/h$, $t_1 = 30\ min$ and $t_2 = 10\ min$

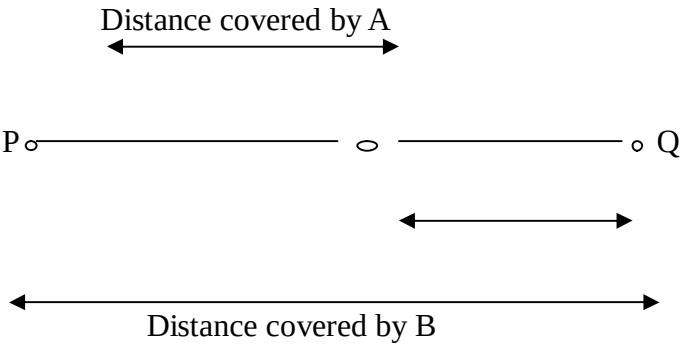
According to the formula,

Required distance $= (t_1 - t_2)(a + b)\frac{a}{b} = \frac{(30-10)}{60}(20 + 4)\frac{20}{4}$

$$= \frac{20}{60} \times 24 \times \frac{20}{4} = 5 \times 8 = 40\ km$$

Technique: 5

When two persons A and B travel from points P to Q, a distance of D with speeds 'a' and 'b', respectively and B reaches Q first, returns immediately and meets A at R, then



Distance travelled by A (from points P to R) $= 2 \times D\left(\frac{a}{a+b}\right)$

Distance travelled by B ($PQ + QR$) $= 2 \times D\left(\frac{b}{a+b}\right)$

Example: 17 Sonu and Monu travel from point P to Q, a distance of 42 km, at 6 km/h and 8 km/h, respectively. Monu reaches Q first and returns immediately and meets Sonu at R. Find the distance from points P to R.

Solution: Given that, $D = 42\ km, a = 6\ km/h$ and $b = 8\ km/h$

Accroding to the formula,

Distance travelled by Sonu = PR $= 2D \times \frac{a}{a+b} = 2 \times 42 \times \frac{6}{6+8}$

$$= 2 \times 42 \times \frac{6}{14} = 2 \times 36 = 36\ km$$

Technique: 6

A policeman sees a thief at a distance of d. He starts chasing the thief who is running at a speed of 'a' and policeman is chasing with a speed of 'b' ($b > a$). In this case, the distance covered by the thief when he is caught by the policeman, is given by $d\left(\frac{a}{b-a}\right)$.

Example: 18 A policeman sees a chain snatcher at a distance of 50 m. He starts chasing the chain snatcher who is running with a speed of $2\ m/s$ while the policeman chasing him with a speed of $4\ m/s$. Find the distance covered by the chain snatcher when he is caught by the policeman.

Solution: Here, $d = 50\ m$, $a = 2\ m/s$ and $b = 4\ m/s$

According to the formula,

Required distance = $d\left(\frac{a}{b-a}\right) = 50 \times \frac{2}{4-2} = 50\ m$.

Technique: 7

Two persons A and B start running at the same time in opposite directions from two points and after passing each other they complete their journeys in 'x' and 'y' h, respectively.

Then, A's speed: B's speed = $\sqrt{y}:\sqrt{x}$

Example: 19 A man sets out to cycle from points P to Q and at the same time another man starts to cycle from points Q to P. After passing each other, they complete their journeys in 9 h and 4 h, respectively. Find the ratio of speeds of 1st man to that of 2nd man.

Solution: Given that, $x = 9\ h$ and $y = 4\ h$

According to the formula,

1st man's speed : 2nd man's speed = $\sqrt{y}:\sqrt{x} = \sqrt{4}:\sqrt{9} = 2:3$

Technique : 8

If a man changes his speed to $\left(\frac{x}{y}\right)$ of his usual speed and gets late by t min or reaches early by t min, then the usual time taken by him

$= \frac{t \times x}{(y-x)}$, if $(y > x)$ and $\frac{t \times x}{(x-y)}$, if $(x > y)$

Example : 20 A man increases his speed to $\frac{7}{5}$ times of his original speed and reaches his office 20 min before to fixed time, then find the usual time taken by him?

Solution : Given that, $\frac{x}{y} = \frac{7}{5}, x = 7, y = 5$ and $t = 20$ min

Now, Required time = $\frac{t \times x}{(x-y)} = \frac{20 \times 7}{(7-5)} = \frac{20 \times 7}{2} = 70$ min