Races

A race or a games of skill includes the contestants in a contest and their skill in the concerned contest/game.

Important Terms:

Race : A race is a contest of speed in running, driving, riding, sailing or rowing.

Race Course : The ground/path on which a contest is organized in a systematic way, is called a race course.

Starting point : The exact point/place from where a race begins, is called starting point.

Start : If two persons A and B are contesting a race and before the start of the race, A is at the starting point and B is ahead of A by 20 m, then it is said that A gives B a start of 20 m.

For example, If A and B are the contestants for a 100 m race and A has to cover 100 m, while B has to cover (100-20)= 80 m.

Finishing point : The exact point/place where a race ends, is known as finishing point.

Winning Point /Goal : A person who reaches the finishing point first, is called the winner.

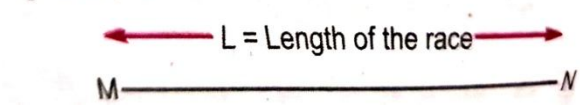
Note: For a winner, finishing point is as same as the winning point/goal.

Dead Heat race : A race is said to be a dead heat race, if all the contestants reach the finishing point exactly at the same time.

Some Facts about Race:

For Two Contestants A and B:

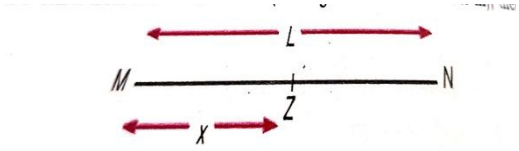
1. If A beats B by x m, then



Distance covered by A (winner) = L m

Distance covered by B (loser) = $(L - x)m$

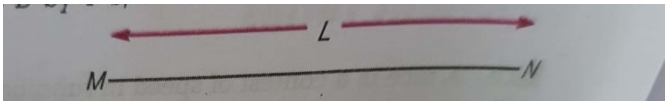
2. If B starts from $x\ m$ ahead of A (or A gives B a start of $x\ m$), then



A starts from M and B starts from Z.

Distance covered by $B = (L - x)m$

3. If A beats B by T s, then

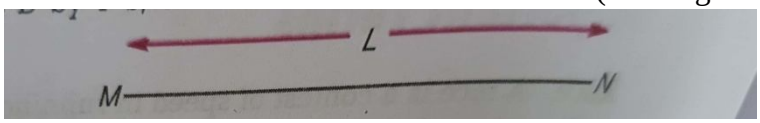


A and B both start from point M.

Time taken by A (winner) = Time taken by B (loser) – T

It means that A completes the race in T s less time than that of B.

4. If B starts the race T s before the time A starts (or if A gives B a start of T s), then



In such case, we say that A starts T s after the time B starts.

5. If both of the contestants get at the finishing point at the same time, then
Difference in time of defeat = 0 ; Difference in distance of defeat = 0

Example : 1 In a race of 100 m, A gives B a start of 10 m. What distance will be covered by B?

Solution: Required distance = (100-10) m = 90 m.

Example : 2 In a race, x gives y a start of 30 m making length of race for y a distance of 170 m. Find the total length of race.

Solution : Required length = (170+30) = 200 m

Example : 3 In a 300 m race, M gives some start to N and this makes the length of race for N 225 m. What start does N get from M?

Solution : Start given by M to N = (300-225) = 75 m.

Example:4 In a 100 m race, Ajay runs at the speed of 4 km/h. Ajay gives Brijesh a start of 4 m and still beats him by 15 s. Find the speed of Brijesh.

Solution : Time taken by Ajay to cover 100 m = $\left(\frac{60\times60}{4000}\times 100\right) s = 90\ s$

Brijesh covers (100 – 4) = 96 m in (90 + 15) = 105 s.

Brijesh's speed = $\frac{96}{105}\ m/s = \frac{96}{105}\times\frac{18}{5} = 3.29\ km/h$

Example : 5 P covers 1 km in 4 min 40 s, while Q covers the same distance in 5 min.By what distance does P defeat Q?

Solution : Clearly, P beats Q by 20 s.

Distance covered by Q in 20 s = $\frac{1000}{300}\times 20 = 66\frac{2}{3}\ m.$

Example : 6 A can run 1 km in 5 min and B can run the same distance in 6 min. How many metres start, can A give to B in 1 km race, so that the race may end in a dead heat?

Solution: Time taken by A to run 1 km = 300 s and Time taken by B to run 1 km = 360 s.

A can give B a start of (360-300) = 60 s

In 360 s, B runs 1000 m.

In 60 s, B runs $\frac{1000}{360}\times 60\ m = \frac{1000}{6}m = \frac{500}{3}m = 166\frac{2}{3}\ m$

Hence, A can give a start of $166\frac{2}{3}\ m.$

Example : 7 In a game of 100 points, A scores 100 points, while B scores only 75 points. In this game, how many points can A give to B?

Solution : Score of A = 100 points and Score of B = 75 points

A can give (100 – 75) = 25 points to B.

Example : 8 In 100 m race, A runs at 8 km/h. If A gives B a start of 4 m and still beats him by 15 s, then what is the speed of B?

Solution: Time taken by A to cover 8 km = 1 h

Time taken by A to cover 8000 m = $60\times 60\ s$

Time taken by A to cover 100 m = $\frac{60\times60}{8000}\times 100 = 45\ s$

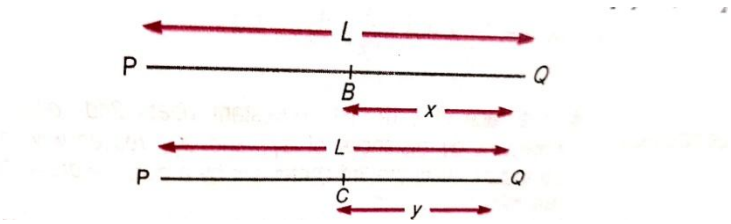
Now, B covers (100-4) = 96 m in (45 + 15) = 60 s

B's speed = $\frac{Distance\ covered}{Time\ taken} = \frac{96}{60}\ m/s = \frac{90\times60\times60}{60\times1000}\ km/h$ = 5.76 km/h.

For Three Contestants A, B and C:

Let A, B and C participate in a race of $L\ m$.

Let A comes 1st in the race by beating B by $x\ m$ and C by $y\ m$, respectively.



Here, the values of x and y will decide 2nd and 3rd positions.

If $x < y$, then B will beat C, ie., B will get the 2nd position.

If $x > y$, then C will beat B, ie., C will get the 2nd position.

Example : 9 A, B and C are three contestants in 1 km race. If A can give B a start of 40 m and A can give C a start of 64 m, how many metres start can B give C?

Solution: While A covers 1000m, B covers (1000-40)= 960 m and

C will cover (1000-64)-936 m.

So, when B covers 1000 m, C will cover $\left(\frac{936}{960} \times 1000\right) = 975\ m$

B can give C a start of (1000-975)m ie., 25 m.

Technique : 1

For 2 contestanrs, following relation will be valid.

$$\frac{Time\ taken\ by\ the\ winner}{Distance\ covered\ by\ the\ loser} = \frac{Time\ taken\ by\ the\ loser}{Distance\ covered\ by\ the\ winner}$$
$$= \frac{Difference\ of\ winning\ time + Initial\ time}{Difference\ of\ winning\ distance + Initial\ distance}$$

Example : 10 In 1 km race, A beats B by 36 m or 18 s. Find the A's time over the course.

Solution : Initial time = 0, Intial distance = 0, difference of winning time = 18 s and difference of winning distance = 36 m

$$\frac{Time\ taken\ by\ the\ winner}{Distance\ covered\ by\ the\ loser} = \frac{Difference\ of\ winning\ time + Initial\ time}{Difference\ of\ winning\ distance + Initial\ distance}$$

$$\frac{Time\ taken\ by\ A}{100-36} = \frac{18}{36}$$

Time taken by A = $\frac{18}{36} \times 964 = \frac{1}{2} \times 964 = 482\ s$

Technique : 2

If in a race of L m, 1st contestant beats 2nd contestant and 3rd contestant by distance of a_{12} and a_{13} respectively and the 2nd contestant beats the 3rd contestant by a distance of a_{23}, then we get the following relation $(L - a_{12})a_{23} = L(a_{13} - a_{12})$

Example : 11 P, Q and R are three contestants in a 2 km race. If P can give Q a start of 100 m and P can give R a start of 138 km, then how many metres start can Q give to R?

Solution : Here $a_{12} = 100\ m, a_{13} = 138\ m, a_{23} =$? and $L = 2000\ m$

According to the formula,

$$(L - a_{12})a_{23} = L(a_{13} - a_{12})$$

$$(2000 - 100)a_{23} = 2000(138 - 100)$$

$$1900a_{23} = 2000 \times 38$$

$$a_{23} = \frac{2000 \times 38}{1900} = \frac{760}{19} = 40\ m$$

Technique : 3

If in a race of $L_1\ m$, 1st contestant beats the 2nd contestant by a distance of a_{12}, in a race of $L_2\ m$ 2nd contestant beats the 3rd contestant by a distance of a_{23} and in a race of $L_3\ m$ 1st contestant beats the 3rd contestant by a distance of a_{13}, then for a race of $L\ m$

$$A_{12} = \frac{a_{12}}{L_1} \times L\ ; A_{23} = \frac{a_{23}}{L_2} \times L;\ \ A_{13} = \frac{a_{13}}{L_3} \times L$$

Now, we get the following relation

$$(L - A_{12})A_{23} = L(A_{13} - A_{12})$$

Example : 12 In a race of 1200 m, A can beat B by 120 m and in race of 500 m, B can beat C by 100 m. By how many metres will A beat C in a race of 800 m?

Solution: If A runs 1200 m, then B runs 1080 m.

If A runs 800 m, then B runs $\left(\frac{1080}{1200} \times 800\right) = 720\ m$

When B runs 500 m, then C runs 400 m.

When B runs 720 m, then C runs $\left(\frac{400}{500} \times 720\right) = 576\ m$

A beats C by $(800 - 576)m = 224\ m$.