Time and Work

In this chapter, we will study technique to solve problems based on work and its completion time as well as number of persons required to finish the given work in stipulated time.

Suppose that you are a contractor and you got a contract to construct a flyover in a certain time. For this, you need to calculate the number of men required to finish the work according to their work efficiency.

Important Relations:

1. **Work and Person:** Directly proportional (more work, more men and conversely more men, more work).
2. **Time and Person:** Inversely proportional (more men, less time and conversely more time, less men).
3. **Work and Time:** Directly proportional (more work, more time and conversely more time, more work).

Note: While solving these types of problems, the work done is always supposed to be equal to 1.

Basic Rules Related to Work and Time:

Rule : 1 If a person can do a piece of work in n days (hours), then that person's 1 day's (hour's) work = $\frac{1}{n}$.

Example : 1 Vandana completes a work in 35 days. What work will she do in 1 day?

Solution : We know that, if a person can do a piece of work in n days, then

Person's 1 day's work = $\frac{1}{n}$

Here, n=35 $\Rightarrow$ Required work done = $\frac{1}{35}$.

Rule : 2 If a person's 1 day's (hour's) work = $\frac{1}{n}$, then the person will complete the work in n days (hours).

Example : 2 Kavi does $\frac{1}{13}$ part of a certain work in 1 day. In how many days, will he complete the whole work?

Solution : We know that, if a person's 1 day's work = $\frac{1}{n}$, then the person will complete the whole work in n days.

Here, $\frac{1}{n} = \frac{1}{13}$

Required number of days = 13.

Rule :3 If a person is n times efficient than the second person, then work done by

First person : Second person = $n:1$

and time taken to complete a work by

First person : Second person = $1:n$

Example : 3 P can do a work 3 times faster than Q and therefore takes 40 days less than Q. Find the time in which P and Q can complete the work individually.

Solution : We know that, if a person is n times efficient than the second person, then time taken to complete a work by

First person : Second person = 1 : n

Time taken to complete the work by P : Q = K : 3K

According to the question, Time taken by Q – Time taken by P = 40

$$3K - K = 40$$

$$2K = 40$$

$$K = 20$$

Number of days required by P = 20 and Number of days required by Q = 60

Rule : 4 If ratio of numbers of men required to complete a work is m : n, then the ratio of time taken by them will be n : m

Example : 4 If 12 men can finish a work in 20 days, then find the number of days required to complete the same work by 15 min.

Solution : We know that, if ratio of numbers of men required to complete a work us m:n, then ratio of time taken by them will be n:m

According to the question, Ratio of numbers of men = 12 : 15 = 4 : 5

Ratio of times taken = 5:4

Let us suppose 15 men can finish a work in x days.

Then, $20:x = 5:4$

Hence, $x = 16$

Required number of days = 16

Fast Track Techniques to solve the Questions

Technique : 1

If M_1 persons can do W_1 work in D_1 days working T_1 h in a day and M_2 persons can do W_2 work in D_2 days working T_2 h in a day, then the relationship between them is

$$M_1D_1T_1W_2 = M_2D_2T_2W_1$$

Example : 5 10 men can make 20 toys in 12 days working 12 h a day. Then, in how many days can 24 persons make 32 toys working 16 h a day?

Solution : Given that, $M_1 = 10, M_2 = 24, D_1 = 12, D_2 = ?, T_1 = 12, T_2 = 1, W_1 = 20$ and $W_2 = 32$

According to the formula,

$$M_1D_1T_1W_2 = M_2D_2T_2W_1$$

$$10 \times 12 \times 12 \times 32 = 24 \times D_2 \times 16 \times 30$$

$D_2 = \frac{10\times12\times12\times32}{24\times16\times20} = \frac{12}{2} = 6$ days.

Technique: 2

(i) If A can do a piece of work in x days and B can do the same work in y days, then $(A + B)'s$ 1 day's work $= \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$

Time taken by (A+B) to complete the work $= \frac{xy}{x+y}\ days$

(ii) If A can do a piece of work in x days and B can do the same work in y days and C can do the same work in z days, then

Time taken by (A+B+C) to complete the work $= \frac{xyz}{xy+yz+zx}\ days$

For n persons, their 1 day's work $= \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots\ldots + \frac{1}{x_n}$

Where $x_1, x_2, x_3, \ldots\ldots, x_n$ represents numbers of days taken by them to complete a work.

Example : 6 A can do a piece of work in 10 days and B can do the same work in 12 days. How long will they take to finish the work, if both work together?

Solution : Here, $x = 10$ and $y = 12$

Number of days taken by A and B = $\frac{xy}{x+y} = \frac{12\times10}{12+10} = 5\frac{5}{11}$ days.

Example : 7 If A can do a piece of work in 4 days, B can do the same work in 8 days and C can do the same work in 12 days, then working together, how many days will they take to complete the work?

Solution : A's 1 day's work = $\frac{1}{4}$, B's 1 day's work = $\frac{1}{8}$, C's 1 day's work = $\frac{1}{12}$

According to the question,

(A+B+C)'s 1 day's work = $\frac{1}{4} + \frac{1}{8} + \frac{1}{12} = \frac{6+3+2}{24} = \frac{11}{24}$

(A+B+C) complete the whole work in $\frac{24}{11}$ days = $2\frac{2}{11}$days.

Technique : 3

If A and B can complete a work in x days and A alone can finish that work in y days, then

Number of days required to complete the work by $B = \frac{xy}{y-x}\ days.$

Example : 8 A and B together can do a piece of work in 12 days and A alone can do it in 18 days. In how many days can B alone do it?

Solution : Here, $x = 12$ and $y = 18$

Time taken by B = $\frac{xy}{y-x} = \frac{12\times18}{18-12} = 36\ days.$

Technique : 4

If A and B can do a piece of work in x days, B and C can do the same work in y days and A and C can do it in z days, then working together A, B and C can do that work in

$$\frac{xyz}{xy + yz + zx}\ days.$$

Example : 9 A and B can do a piece of work in 3 days. B and C can do the same work in 9 days, while C and A can do it in 12 days. Find the time in which A, B and C can finish the work, working together.

Solution : Here, $x = 3, y = 9, z = 12$

According to the formula,

Required time taken by A, B and C = $\frac{2xyz}{xy+yz+zx}$

$$= \frac{2 \times 3 \times 9 \times 12}{3 \times 9 + 9 \times 12 + 3 \times 12} = \frac{2 \times 3 \times 9 \times 12}{27 + 108 + 36} = \frac{2 \times 3 \times 9 \times 12}{171}$$

$= \frac{6\times12}{19} = \frac{72}{19} = 3\frac{15}{19}$ days.

Technique : 5

If a_1 men or b_1 women can finish a work in D days, then time taken by a_2 men and b_2 women to complete the work = $\frac{D(a_1b_1)}{(a_2b_1+a_1b_2)}$ $days$.

Example :10 If 6 men or 8 women can reap a field in 86 days, how long will 14 men and 10 women take to reap it?

Solution : Here, $a_1 = 6, b_1 = 8, a_2 = 14, b_2 = 10$ and $D = 86$

Number of days = $\frac{D(a_1b_1)}{(a_2b_1+a_1b_2)} = \frac{86\times6\times8}{14\times8+10\times6} = \frac{86\times6\times8}{172} = 24\ days$.

Technique : 6

If A can do a work in x days and B can do y% fast than A, then B will complete the work in $\frac{100x}{100+y}$ $days$.

Example : 11 Kamal can do a work in 15 days and Vimal is 50% more expert than Kamal to complete the same work, then find total time taken to complete the work by Vimal.

Solution : Here, $x = 15\ days\ and\ y = 50\%$

Now, time taken by Vimal = $\frac{100x}{100+y} = \frac{100\times15}{100+50} = \frac{1500}{150} = 10\ days$.

Technique : 7

If a men can do a piece of work in x days and b boys can do the same work in y days, then time taken to complete the same work by c men and d boys will be $\frac{1}{\frac{c}{ax}+\frac{d}{by}}$ days.

Example : 12 If 5 men can do a work in 2 days and 3 boys can do the same work in 5 days, then find the time taken to complete same work by 10 men and 3 boys.

Solution : Given, $a = 5, b = 3, x = 2, y = 5, c = 10$ and $d = 3$

Time taken by 10 men and 3 boys = $\frac{1}{\frac{c}{ax}+\frac{d}{by}} = \frac{1}{\frac{10}{5\times2}+\frac{3}{3\times5}} = \frac{1}{1+\frac{1}{5}} = \frac{5}{6}$ days.

Technique : 8

If a_1 men and b_1 boys can complete a work in x days, while a_2 men and b_2 boys can complete the same work in y days, then

$$\frac{One\ day\ work\ of\ 1\ man}{One\ day\ work\ of\ 1\ boy} = \frac{(yb_2 - xb_1)}{(xa_1 - ya_2)}$$

Example : 13 If 12 men and 16 boys can finish a work in 5 days, while 13 men and 24 boys can finish the same work in 4 days. Compare the one day work of 1 man and 1 boy.

Solution : Here, $a_1 = 12, b_1 = 16, x = 5, a_2 = 13, b_2 = 24$ and $y = 4$

$$\frac{One\ day\ work\ of\ 1\ man}{One\ day\ work\ of\ 1\ boy} = \frac{(yb_2 - xb_1)}{(xa_1 - ya_2)} = \frac{4 \times 24 - 5 \times 16}{5 \times 12 - 4 \times 13}$$

$$= \frac{96 - 80}{60 - 52} = \frac{16}{8} = \frac{2}{1} = 2$$

Technique : 9

If x takes a days more to complete a work than the time taken by $(x + y)$ to do same work and y takes b days more than the time taken by $(x + y)$ to do the same work, then $(x + y)$ do the work in $\sqrt{ab}$ days.

Example : 14 When A alone does a piece of work, he takes 16 days more than the time taken by (A+B) to complete the work, while B alone takes 9 days more than the time taken by (A+B) to finish the work. What time, A and B together will take to finish the work?

Solution : Here, $a = 16$ and $b = 9$

According to the formula, Required time = $\sqrt{ab}$

Required time = $\sqrt{16 \times 9} = 4 \times 3 = 12$ days

Technique : 10

A and B, each alone can do a piece of work in a and b days, respectively.

Both begin together and if

(i) A leaves the work x days before its completion, then total time taken for completion of work will be given as $T = \frac{(a+x)b}{(a+b)}$ days.

(ii) B leaves the work x days before its completion, then total time taken for completion of work will be given as $T = \frac{(b+x)a}{(a+b)}$ days.

Example : 15 A can do a piece of work in 10 days while B can do it in 15 days. They begin together but 5 days before the completion of the work, B leaves off. Find the total number of days for the work to be completed.

Solution : Here, $a = 10\ days, b = 15\ days, x = 5\ and\ T =?$

According to the formula,

Required time $= \frac{(b+x)a}{(a+b)} = \frac{(15+5)10}{10+15} = \frac{20\times10}{25} = 4 \times 2 = 8$ days.

Technique : 11

A and B do a piece of work in a and b days, respectively. Both begin together but after some days, A leaves off and the remaining work is completed by B in x days. Then, the time after which A left, is given by

$$T = \frac{(b-x)a}{a+b}$$

Example : 16 A and B can do a piece of work in 40 days and 50 days, respectively. Both begin together but after a certain time. A leaves off. In this case B finishes the remaining work in 20 days. After how many days did A leave?

Solution : Here, $a = 40\ days, b = 50\ days, x = 20$ and $T =?$

Required time $= \frac{(b-x)a}{a+b} = \frac{(50-20)\times40}{(40+50)} = \frac{30\times40}{90} = \frac{40}{3} = 13\frac{1}{3}$ days.