

PART-B**UNIT 1, PROBABILITY AND RANDOM VARIABLES**

1. A random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find

- (1) The value of k
- (2) Evaluate $P(X < 6), P(0 < X < 5)$
- (3) The smallest value of a for which $P(X \leq a) > \frac{1}{2}$.
- (4) The Cumulative distribution function.

2. A random variable X has the following probability function

X	0	1	2	3	4	5	6	7	8
$P(x)$	A	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Find (i) Determine the value of 'a'

(ii) Find $P(X < 3), P(X \geq 3), P(0 < X < 5)$

(iii) Find the distribution function of X .

3. A random variable X has the following probability distribution

X	-2	-1	0	1	2	3
$P(x)$	0.1	K	0.2	$2K$	0.3	$3K$

Find (1) The value of K (2) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$ (3)

Find the Cumulative distribution of X (4) Find the mean of X .

4. If the Random variable X takes the value 1,2,3,4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$. Find the probability distribution.
5. A continuous R.V X has the p.d.f $f(x) = 3x^2, 0 \leq x \leq 1$. Find the value of a , such that $P(X \leq a) = P(X > a)$. Find the value b such that $P(X > b) = 0.05$.

6. A continuous R.V. X has the p.d.f. $f(x) =$

$$\begin{cases} \frac{k}{1+x^2} & -\infty < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find

- (1) The value of k
- (2) Distribution function of X
- (3) $P(X \geq 0)$

7. The probability function of an infinite discrete distribution

is given by $P(X = j) = \frac{1}{2^j}$ ($x = 1, 2, 3, \dots$)

- (1) Mean and variance of X
- (2) $M.G.F$
- (3) $P(X \text{ is even})$

8. A random variable has the pdf $f(x) =$

$$\begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

. Obtain the mgf and first four moments about the origin. Also find the mean and variance.

9. Find the M.G.F of the random variable with the probability law $P(X = x) = q^{x-1}p$, $x = 1, 2, 3, \dots$. Find the mean and variance.

10. A continuous Random variable X has the distribution

$$\text{function } F(x) = \begin{cases} 0 & x \leq 1 \\ k(x-1)^4 & 1 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$$

- (1) Find K
- (2) p.d.f $f(x)$
- (3) $P(X < 2)$.

11. The diameter of an electric cable say X , is assumed to be a continuous Random variable with P.d.f

$$f(x) = 6x(1 - x) , 0 \leq x \leq 1$$

- (i) Check that the above is a P.d.f
- (ii) Determine a and b such that $P(X < b) = P(X > b)$
- (iii) Find the distribution function of X
- (iv) Find $P(X \leq \frac{1}{2} / \frac{1}{3} < X < \frac{2}{3})$

12. If the probability density of X is given by

$$f(x) = \begin{cases} 2(1 - x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find its r th moment. Hence evaluate $E[(2x + 1)^2]$

13. If the cumulative distribution function of X is given

$$\text{by } F(x) = \begin{cases} 1 - \frac{4}{x^2}, & x > 2 \\ 0 & , x \leq 2 \end{cases}$$

Find (i) $P(X < 3)$ (ii) $P(4 < X < 5)$ (iii) $P(X \geq 3)$.

14. Experience has shown that walking in a certain park, the time X (in mins) , between seeing two people smoking has a density function of the form $f(x) =$

$$\begin{cases} \lambda x e^{-x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Calculate the value of λ
- (b) Find the distribution function of X
- (c) What is the probability that a person who has just seen a person smoking will see another person smoking in 2 to 5 minutes? In atleast 7 minutes?

15. The density function of a random variable X is given

$$\text{by } f(x) = \begin{cases} kx(2 - x)^2 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases} \text{ find (i) } k \text{ (ii) Mean}$$

and variance of the distribution.
