SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

Consider,

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n y = f(x) \longrightarrow ①$$

where $a_0, a_1, a_2, \ldots a_n$ are constants and $f(x)$ is a function of x is called a homogeneous linear differential equation of order n with variable coefficients.

For this type, substitute

$$z = \log x \quad (or) \quad x = e^z$$

Then

$$x \frac{dy}{dx} = D'y$$

$$x^2 \frac{d^2y}{dx^2} = D'(D'-1)y$$

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DEPATMENT OF MATHEMATICS

$$x^3 \frac{d^3y}{dx^3} = D'(D'-1)(D'-2)y \quad \text{and so on.}$$

Problems :

(1) Transform the equation to constant coefficient equations :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \, \sin(\log x).$$

Soln :

Replace $\log x = z$

$$x\frac{dy}{dx} = D'y$$

$$x^2 \frac{d^2y}{dx^2} = D'(D'-1)y$$

Given equation becomes,

$$D'(D'-1)y + D'y + y = z \sin z$$

$$(D'^2 - D')y + D'y + y = z \sin z$$

$$(D'^2 - \cancel{D'} + \cancel{D'} + 1)y = z \sin z$$

$$\boxed{(D'^2 + 1)y = z \sin z}$$